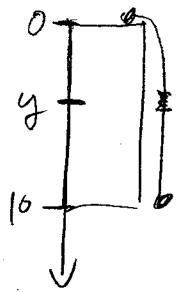
Quiz 2 25 points Math 160 Name: ________________________

1. (4 points) Find the work done pulling a 50 kg mass from the ground to the top of a 10 meter building using a chain with density 3 kg/meter. Use g for standard gravity.



$$W = \int_0^{10} (50 + 3y) g \, dy$$

$$= 50 \cdot 10 g + \left(\frac{3y^2 g}{2} \right) \Big|_0^{10}$$

$$= 500g + 150g = \boxed{650g \text{ Joules}}$$

$D_y = \Delta y$

$F_y = (50 + 3y) g$

2. (4 points) An ten Newton force is required to hold a spring at 2 meters from equilibrium. How much work is needed to stretch the spring from equilibrium to 5 meters beyond it?

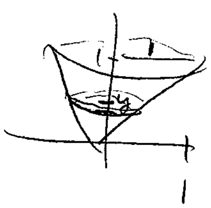
$10 = k \cdot 2 \Rightarrow k = 5$

$$W = \int_0^5 5x \, dx = \frac{5x^2}{2} \Big|_0^5 = \boxed{\frac{125}{2} \text{ Joules}}$$

$F_x = 5x$

$D_x = dx$

3. (5 points) Find the work done pumping water out the top of a conical tank formed by rotating the curve $y = x$ about the y-axis, $0 \leq x \leq 1$. Use ρ for the density of water and g for standard gravity.



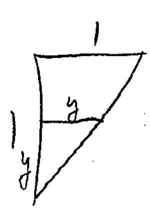
$$W = \int_0^1 \pi y^2 \rho g (1 - y) \, dy$$

$$= \pi \rho g \left(\frac{y^3}{3} - \frac{y^4}{4} \right) \Big|_0^1$$

$$= \boxed{\frac{\pi \rho g}{12} \text{ Joules}}$$

$D_y = 1 - y$

$F_y = \pi y^2 \cdot \rho g \, dy$



Q2

4. Evaluate the following integrals. Show work or some defense of your answer.

(a) (4 points) $I = \int_0^{\pi} e^{\cos(t)} \sin(2t)\, dt$ $= \int_0^{\pi} e^{\cos(t)} \cdot 2\sin(t)\cos(t)\, dt$

$u = \cos(t)$
$du = -\sin(t)\, dt$ $\Rightarrow$ $I = 2\int_1^{-1} e^u u (-1)\, du$

	u	e^u
$\oplus$	u	e^u
$\ominus$	1	e^u
	0	e^u

$\Rightarrow I = 2\left[ue^u - e^u\right]_{-1}^{1} = 2\left[(e-e) - (-e^{-1} - e^{-1})\right]$

$\Rightarrow \boxed{I = 4e^{-1}}$

(b) (4 points) $I = \int_0^{0.5} x\sqrt{1-4x^2}\, dx$

$u = 1-4x^2$
$du = -8x\, dx$ $\Rightarrow I = \int_1^0 -\frac{1}{8}\sqrt{u}\, du$

$= \frac{1}{8}\int_0^1 u^{1/2}\, du$

$= \frac{1}{8} \cdot \frac{2}{3} u^{3/2} \Big|_0^1 = \boxed{\frac{1}{12}}$

(c) (4 points) $I = \int \frac{dx}{1-\sin(x)}$. Hint: Conjugate!

$I = \int \frac{1+\sin(x)}{1-\sin^2 x}\, dx = \int \frac{1}{\cos^2(x)} + \frac{\sin(x)}{\cos^2(x)}\, dx$

$= \int \sec^2(x) + \sec(x)\tan(x)\, dx$

$= \boxed{\tan(x) + \sec(x) + C}$