Quiz 3 25 points Math 160 Name: ______________________

1. (5 points) Evaluate $I = \int x^2 \sinh(x+2)\, dx$.

$+$	x^2	$\sinh(x+2)$
$-$	$2x$	$\cosh(x+2)$
$+$	2	$\sinh(x+2)$
$-$	0	$\cosh(x+2)$

$$\Rightarrow \boxed{I = x^2\cosh(x+2) - 2x\sinh(x+2) + 2\cosh(x+2) + C}$$

2. (5 points) Is $I = \int_0^1 x^{-3/2}\, dx$ convergent or divergent? Evaluate if convergent, explain why if divergent.

$$I = \lim_{a\to 0^+} \int_a^1 x^{-\frac{3}{2}}\, dx = \lim_{a\to 0^+} \left(-2x^{-\frac{1}{2}}\right)\Big|_a^1$$

$$= \lim_{a\to 0^+} -2\left[1 - a^{-\frac{1}{2}}\right] = -2(1-\infty) = \infty.$$

$$\boxed{\therefore\ I \text{ is divergent.}}$$

3. (5 points) Does $I = \int_0^\infty \frac{1}{2+e^x}\, dx$ converge or diverge? Defend your answer completely. (Hint: compare!) $2+e^x > e^x$

$$I = \lim_{b\to\infty} \int_0^b (2+e^x)^{-1}\, dx < \lim_{b\to\infty} \int_0^b e^{-x}\, dx$$

$$= \lim_{b\to\infty} \left(-e^{-x}\right)\Big|_0^b$$

$$= \lim_{b\to\infty} \left(1 - e^{-b}\right)$$

$$= 1 - 0 = 1.$$

Since $\frac{1}{2+e^x} > 0$, Comparison $\Rightarrow$ $\boxed{I \text{ converges}}$

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4. Evaluate the following.

(a) (5 points) $I = \int \frac{3x+6}{x(x+1)(x-2)}\,dx$

$$I = \int \frac{-3}{x} + \frac{+1}{x+1} + \frac{2}{x-2}\,dx$$

$$= \boxed{-3\ln|x| + \ln|x+1| + 2\ln|x-2| + C}$$

Scratch

Use cover-up

$$\left.\frac{3x+6}{x(x+1)(x-2)}\cdot x\right|_{x=0} = \frac{6}{-2}$$

$$\left.\frac{3x+6}{x(x+1)(x-2)}\cdot(x+1)\right|_{x=-1} = \frac{3}{+3}$$

$$\left.\frac{3x+6}{x(x+1)(x-2)}\cdot(x-2)\right|_{x=2} = \frac{12}{6}$$

OR

Partial fractions:

$3x+6 = A(x+1)(x-2) + Bx(x-2) + Cx(x+1)$

$\Rightarrow A+B+C=0$
$-A-2B=3$
$-2A-C=6$; solve, etc.

(b) (5 points) $I = \int_0^3 \frac{dx}{\sqrt{x^2+9}}$. Simplify your answer.

$\cos^2 x + \sin^2 x = 1 \overset{\div\cos^2 x}{\Rightarrow} 1 + \tan^2 x = \sec^2 x.$

Let $x = 3\tan\theta$.
$dx = 3\sec^2\theta\,d\theta$

$x=0 \Rightarrow \theta=0$
$x=3 \Rightarrow \theta=\pi/4$

$$\Rightarrow I = \int_0^{\pi/4} \frac{3\sec^2\theta\,d\theta}{\sqrt{9\tan^2\theta+9}}$$

$$= \int_0^{\pi/4} \frac{3\sec^2\theta\,d\theta}{3\sec\theta}$$

$$\Rightarrow I = \int_0^{\pi/4} \sec\theta\,d\theta = \ln|\sec\theta + \tan\theta|\Big|_0^{\pi/4}$$

$$= \ln\left|\frac{2}{\sqrt{2}}+1\right| - \ln|1-0|$$

$$= \boxed{\ln(\sqrt{2}+1)}$$