Quiz 5 10 points Math 160 Name: ______________________

1. (3 points) Solve $(x^2+1)\dfrac{dy}{dx} = e^{-y}$ implicitly if $y(0) = 0$.

$$\Rightarrow \int e^y \, dy = \int \frac{dx}{x^2+1}$$

$$\Rightarrow e^y = \arctan(x) + C.$$

$$y(0)=0 \Rightarrow 1 = 0 + C$$

$$\boxed{\therefore\ e^y = \arctan(x) + 1}$$ is an implicit solution.

2. (3 points) Find the explicit general solution for $x\dfrac{dy}{dx} - 2y = -x^3$. Remember to put the equation in standard form first.

$$\Rightarrow y' - \frac{2}{x} y = -x^2$$

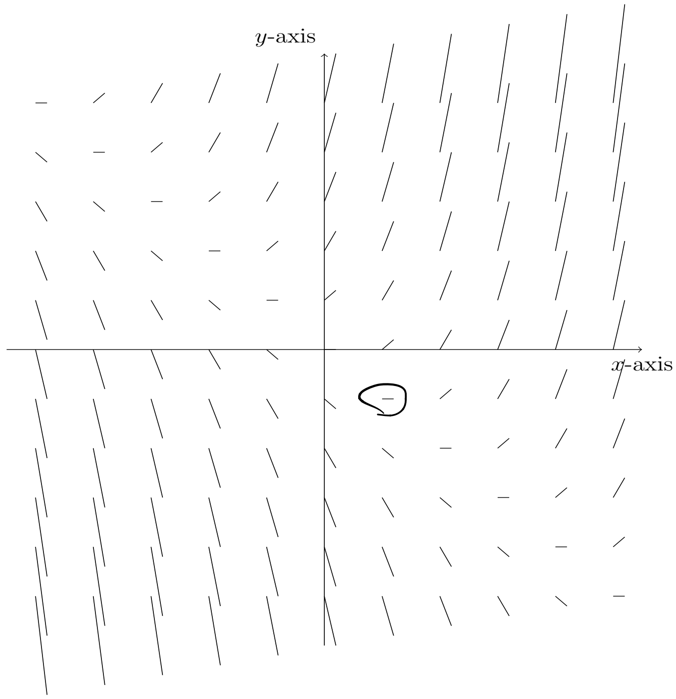
$$u = e^{\int -\frac{2}{x} dx} = e^{-2\ln(x)} = x^{-2}$$

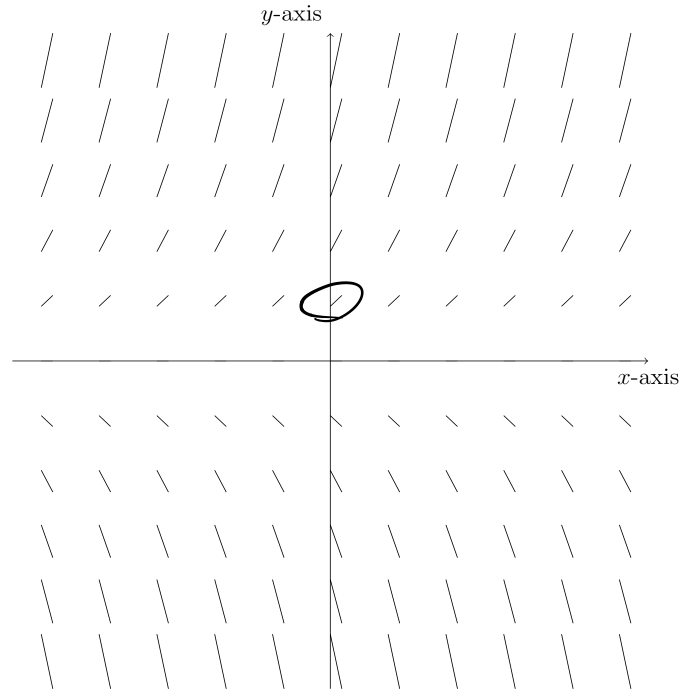
$$\xrightarrow{\cdot u} \frac{d(x^{-2}y)}{dx} = -1$$

$$\xrightarrow{\int dx} x^{-2}y = -x + C$$

$$\Rightarrow \boxed{y = -x^3 + Cx^2}$$

3. (2 points) Two of the slope fields for $y' = 0.1yx$, $y' = y + x$, and $y' = y$ are given below. Which one is missing? Defend your answer completely.





This slope field has a zero slope when x and y are both nonzero, so it cannot be $\underline{y' = 0.1yx}$ or $y' = y$.

This slope field is not zero when $x = 0$ and $y > 0$, so it cannot be $\underline{y' = 0.1yx}$.

$\boxed{\therefore \text{ neither can be } y' = 0.1yx}$

4. (1 point) A liquid in a room kept at a temperature of 22°C cools at a rate equal to half of $T(t) - 22$ where $T(t)$ is the temperature at time t. Write down the differential equation for $T(t)$ and find its general solution by inspection.

$$\frac{dT}{dt} = -\frac{1}{2}(T - 22)$$

$$\frac{d(T-22)}{dt} = \frac{dT}{dt}$$

$$\Rightarrow \boxed{T - 22 = Ce^{-\frac{1}{2}t}} \text{ or } \boxed{T = 22 + Ce^{-\frac{1}{2}t}}$$

5. (1 point) A lake has a carrying capacity of 1000 fish. Assume the fish population grows logistically with growth rate $k = 0.2$ day^{-1} and that there are 10 fish at time zero. Set up an initial value problem for $P(t)$, the population at time t, **but do not solve it.**

$$\boxed{\frac{dP}{dt} = 0.2P\left(1 - \frac{P}{1000}\right) \; ; \; P(0) = 10}$$

Malthusian: $\frac{dP}{dt} = kP$

Logistic: $\frac{dP}{dt} = k\left(1 - \frac{P}{C}\right)P$