Quiz 8 25 points Math 160 Name: ______________________

1. (3 points) Find a position function for the line passing through the points $P = (1, 2, 3)$ and $Q = (3, 1, 2)$. Write your answer as a single vector.

$\vec{QP} = \langle 1,2,3\rangle - \langle 3,1,2\rangle = \langle -2,1,1\rangle$ is the direction vector.

$\vec{p}(t) = \langle 1,2,3\rangle + t\langle -2,1,1\rangle \Rightarrow \boxed{\vec{p}(t) = \langle 1-2t, 1+t, 3+t\rangle}$

other equivalent answers:

$\vec{p}(t) = \langle 3-2t, 1+t, 2+t\rangle$

$\vec{p}(t) = \langle 1+2t, 2-t, 3-t\rangle$

$\vec{p}(t) = \langle 3+2t, 1-t, 2-t\rangle$

2. Find the following if $\vec{v} = 3\hat{\imath} - 2\hat{\jmath} + \hat{k}$ and $\vec{w} = \langle -1, 4, 2\rangle$.

(a) (2 points) $\vec{v} \cdot \vec{w}$

$\vec{v}\cdot\vec{w} = 3(-1) + (-2)(4) + (1)(2) = -3 - 8 + 2 = \boxed{-9}$

(b) (2 points) The work done by a force $\vec{v}$ Newtons on a particle with displacement $\vec{w}$ meters.

$\vec{v}\cdot\vec{w} = \boxed{-9}$ Joules

(c) (2 points) $\cos(\theta)$ if θ is the angle between $\vec{v}$ and $\vec{w}$.

$\vec{v}\cdot\vec{w} = \|\vec{v}\|\,\|\vec{w}\|\cos\theta \Rightarrow -9 = \sqrt{9+4+1}\,\sqrt{1+16+4}\,\cos\theta$

$\Rightarrow \boxed{\cos\theta = \frac{-9}{\sqrt{14}\sqrt{21}} = \frac{-9}{7\sqrt{6}}}$ or $\frac{-3\sqrt{6}}{14}$

(d) (2 points) The projection of $\vec{v}$ onto $\vec{w}$.

$\vec{v}_{\parallel\vec{w}} = \frac{\vec{v}\cdot\vec{w}}{\vec{w}\cdot\vec{w}}\vec{w} = \frac{-9}{21}\vec{w} = \boxed{-\frac{3}{7}\langle -1,4,2\rangle}$

(e) (2 points) The equation of the plane in standard form that is perpendicular to $\vec{w}$ and passes through the point $(1, 2, 5)$.

$\langle -1,4,2\rangle\cdot\langle x,y,z\rangle = \langle -1,4,2\rangle\cdot\langle 1,2,5\rangle$

$\Rightarrow -x + 4y + 2z = -1 + 8 + 10 \Rightarrow \boxed{-x + 4y + 2z = 17}$

Q8

3. Find the following if $\vec{u} = 2\hat{\imath} + 2\hat{\jmath} + \hat{k}$ and $\vec{p} = < 2, -1, 1 >$.

(a) (4 points) $\vec{p} \times \vec{u}$

$$\vec{p}\times\vec{u} \quad \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -1 & 1 \\ 2 & 2 & 1 \end{vmatrix} = (-1-2)\hat{i} - (2-2)\hat{j} + (4+2)\hat{k} = \boxed{\langle -3, 0, 6 \rangle}$$

(b) (2 points) $\vec{u} \times \vec{p} \cdot < 1, 1, 1 >$

$$(-1)\langle -3, 0, 6\rangle \cdot \langle 1,1,1\rangle = \boxed{-3}$$

Since $\vec{u}\times\vec{p} = -\vec{p}\times\vec{u}$.

(c) (2 points) The area of the triangle determined by $\vec{p}$ and $\vec{u}$

$$\text{Area triangle} = \frac{1}{2}\|\vec{u}\times\vec{p}\| = \frac{1}{2}\sqrt{9+36} = \boxed{\frac{3}{2}\sqrt{5}}$$

4. (4 points) Find an equation for the plane in standard form that contains the points $\overset{P=}{(1, 0, 1)}$, $\overset{Q=}{(1, 1, 0)}$, and $\overset{R=}{(0, 1, 1)}$.

$$\overrightarrow{PQ} = \langle 1,1,0\rangle - \langle 1,0,1\rangle = \langle 0,1,-1\rangle$$
$$\times\ \overrightarrow{PR} = \langle 0,1,1\rangle - \langle 1,0,1\rangle = \langle -1,1,0\rangle$$
$$\langle 1,1,1\rangle$$

$$\Rightarrow \langle 1,1,1\rangle \cdot \langle x,y,z\rangle = \langle 1,1,1\rangle\cdot\langle 1,0,1\rangle$$

$$\Rightarrow \boxed{x+y+z=2}$$