

Always show work to defend your answer in a logical and organized fashion unless told otherwise.

1. (15 points) Find the volume of a solid with the unit disk $x^2 + y^2 \leq 1$ for a base and with triangular cross sections perpendicular to the x -axis that have equal height and base lengths.

2. (10 points) Use a trigonometric substitution to evaluate $I = \int_0^1 \frac{x^2}{\sqrt{4-x^2}} dx$.

3. (15 points) First use trigonometric identities to change the integrand and then find antiderivatives while evaluating $I = \int \cos(7x) \sin(4x) + \cos^2(3x) dx$.

4. (10 points) Use integration by parts to evaluate $I = \int_1^e x^2 \ln(x) dx$.

5. (15 points) Find the work done pumping water out from a full concave-up 4-meter radius hemispherical tank through a spout on top that is 1 meter high. Use 1000 kg/m^3 for the density of water and 10 m/sec^2 for standard gravity.

6. (10 points) What is the average value of $f(x) = x^2 \cos(x)$ over the interval $[0, \pi]$?

7. (15 points) The region R is bounded by $y = \frac{9}{x}$ and $y = 10 - x$. Sketch the region R and then find two integrals equal to the volume of the solid formed by rotating R about $y = 0$. The first integral must use the x -axis as the indexing axis and the second must use the y -axis for the indexing axis. Then find the volume by evaluating one of the two integrals.

8. (10 points) The density of gorillas per square kilometer is $\rho(x) = 52(1 + x^2)^{-1}$ where x is the distance in kilometers from a water source. Use an integral to calculate the number of gorillas within a 5 kilometer circle about the water source.