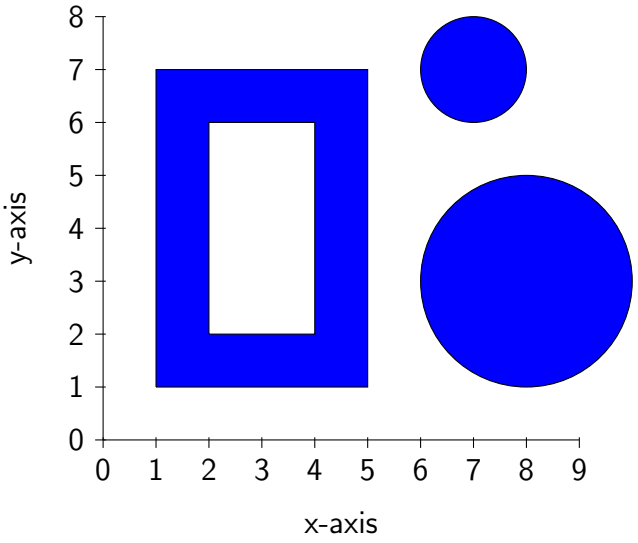
Test 2 B 100 points Math 160 Name: _____

1. (6 points) Find the center of mass for the one-dimensional metal rod with linear density $\delta(x) = \sin(x^2)$ for $0 \leq x \leq \sqrt{\pi}$.

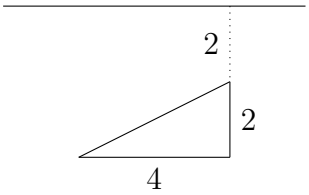
2. (6 points) Solve $y' = 2y + 8, y(0) = 1$ by inspection using only a tiny bit of algebra.

3. (6 points) $S = \sum_{n=0}^{\infty} \frac{2^n}{\pi^n}$ converges. Find the sum.

4. (6 points) Find the centroid for the shaded region. Indicate how you found your answer.



5. (10 points) Use an integral representation to calculate the hydrostatic force on one side of a right triangular plate with legs of two and four meters submerged vertically two meters below the surface of the water as shown in the picture. Use ρ kg/m^3 for density of water and g m/sec^2 for standard gravity.



6. (8 points) Solve $e^x \dfrac{dy}{dx} + 2e^x y = x$ if $y(0) = 5$. Solve for y explicitly.

7. (10 points) Use the Divergence Test or the integral test to determine convergence or divergence of the following series.

   (a) $S = \displaystyle\sum_{n=1}^{\infty} \frac{n}{e^{n^2}}$

   (b) $S = \displaystyle\sum_{n=2}^{\infty} \frac{n^2 + 2}{n^2 \cos(n\pi) + 4}$

8. (8 points) Solve $\dfrac{dy}{dx} = \dfrac{1+y^2}{(x-2)(x+3)}$ for $y(x)$. Find the general solution for y **implicitly**.

9. (8 points) Find the arc length of the curve $y = \ln(\sec(x))$ for $\dfrac{-\pi}{4} \leq x \leq \dfrac{\pi}{4}$.

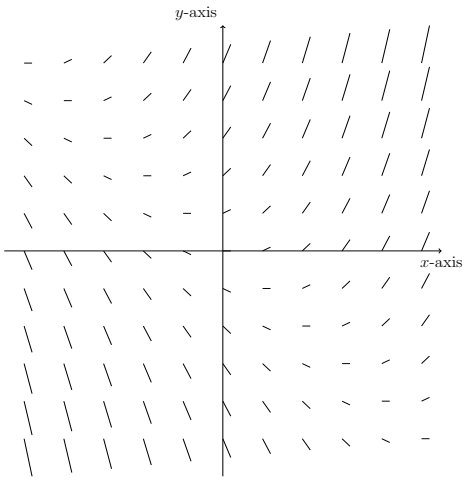
10. Use the direct comparison test or the limit comparison test to determine convergence or divergence of the following series.

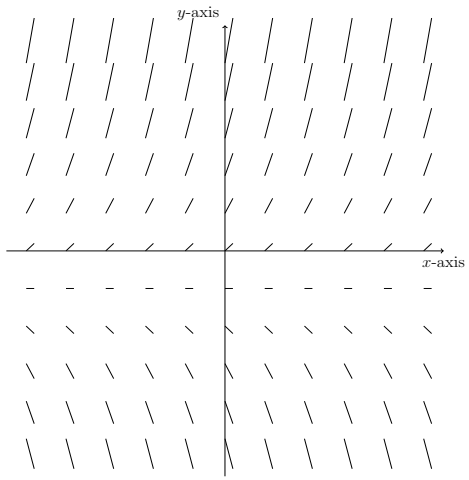
(a) (6 points) $S = \sum_{n=2}^{\infty} \frac{n}{(n^8+2)^{1/4}}$

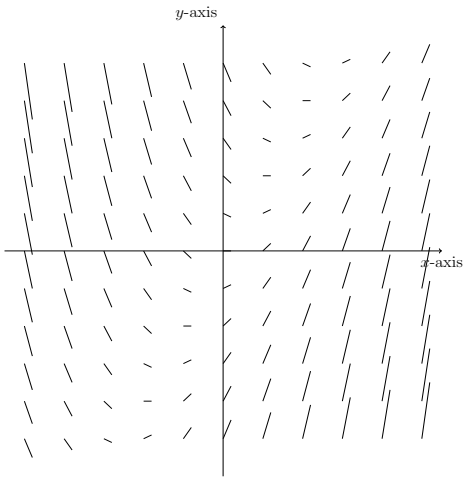
(b) (6 points) $S = \sum_{n=2}^{\infty} \frac{n^3}{n^5+3n+6}$

11. (6 points) Does S $= \sum_{n=4}^{\infty} \frac{(-1)^n}{3n^3+4}$ converge absolutely, converge conditionally, or diverge? Defend your answer.

12. (6 points) The slope fields for $y' = x - 0.5y$, $y' = y + 1$, and $y' = 0.5(x + y)$ are given below. Write the differential equation under its corresponding slope field and defend your answers.







13. (8 points) Use an integral to find the surface area of the solid formed by rotating $y = 2\sqrt{x}$ with $0 \leq x \leq 3$ about the x-axis.