Test 3 100 points Math 160 Name: _____

1. Write the following in simplest form. Show one step of work before writing an answer.

(a) (3 points) $\sum_{n=0}^{\infty}(-1)^n\frac{\pi^{2n}}{(2n)!}$

(b) (3 points) $e^{(5\pi/4)i}$

(c) (3 points) $\sum_{n=0}^{\infty}(-1)^n\frac{\pi^n}{4^n}$

2. (10 points) Find the interval of convergence for S=$\sum_{n=1}^{\infty}\frac{(x+2)^n}{n^2\,8^n}$. Defend your answer.

3. (6 points) Convert $r^2 = 2r\cos(\theta) + 4r\sin(\theta)$ to rectangular coordinates. Use your answer to describe or graph the curve.

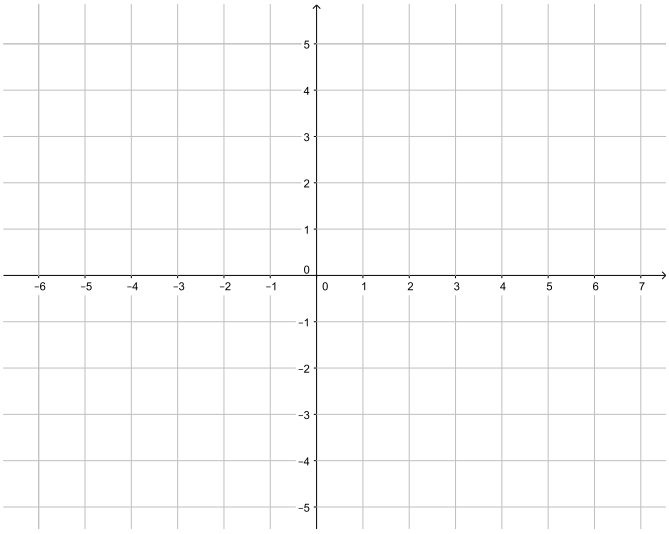
4. (a) (8 points) What is the **velocity** and **speed** at $t = 2$ for a particle with position function $\vec{p}(t) = e^{2t}\langle t^2, t\rangle$?

(b) (4 points) Find the equation of the tangent line to the trace of $\vec{p}(t) = e^{2t}\langle t^2, t\rangle$ at $t = 2$.

5. This question is about the parametric equations $x = \cos(t), y = 1 - 2\sin(t)$.

   (a) (3 points) Convert the parametric equations to a single equation in terms of the rectangular coordinates x and y.

   (b) (3 points) Sketch the trace of the parametric equations if $0 \leq t \leq 2\pi$. The trace includes an arrow in the direction of increasing parameter.



6. (10 points) Find an equation for the plane in standard form that contains the point $P = (1, 3, 1)$ and is parallel to the vectors $\vec{v} = \langle 1, 1, 1 \rangle$ and $\vec{w} = \hat{\imath} + \hat{\jmath} - \hat{k}$

7. (10 points) Use the definition (and differentiation) to find the degree two Taylor polynomial for $f(x) = \sec(x)$ expanded about $x = \pi$, and then estimate $I = \int_{\pi-1}^{\pi+1} f(x)\,dx$ using it.

8. (6 points) Find a position function for the line passing through the points $P = (2, -3, 1)$ and $Q = (4, 1, 1)$. Write your function as a single vector.

9. Let $\vec{x} = 2\hat{\imath} + 4\hat{\jmath} - 5\hat{k}$ and $\vec{y} =< 2, 2, -1 >$. Calculate the following. Simplify your final answer.

   (a) (5 points) $5\vec{x} - \vec{x} \times 2\vec{y}$

   (b) (5 points) $||\vec{y}|| + (\vec{y} \times \vec{x}) \cdot \vec{y} + < 1, 1, 1 > \cdot \vec{x}$

10. (8 points) Find the area of the region that is inside the circle $r = 4\cos(\theta)$ and outside the circle $r = 2$ in the xy-plane.

11. Find the following if $\vec{v} =< 1, 1, -2 >$ and $\vec{w} = 2\hat{\imath} + \hat{\jmath} - 2\hat{k}$.

(a) (3 points) $\vec{w}_{||\vec{v}}$

(b) (3 points) $\cos(\theta)$ if θ is the angle between $\vec{v}$ and $\vec{w}$.

12. (7 points) Find the arc length for the trace of $\vec{p}(t) = \langle \cosh(t), 5 - t \rangle$ for $0 \leq t \leq 1$.