Test 3 100 points Math 160 Name: _____

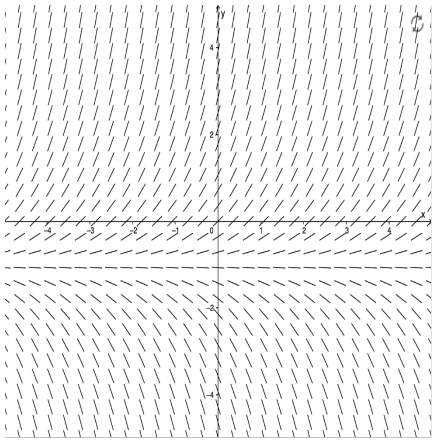
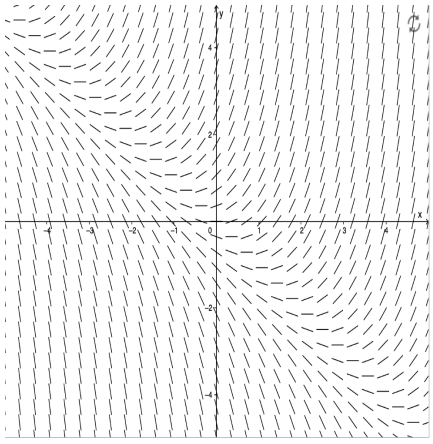
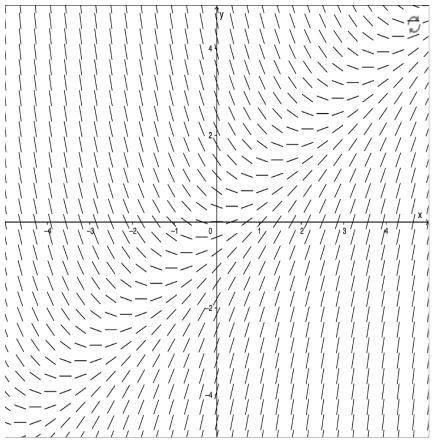
Always show work to defend your answer in a logical and organized fashion unless told otherwise.

1. (10 points) Find the explicit solution to $\frac{dy}{dx} + 4xy = 4x^3e^{-2x^2}$ if $y(0) = -1$.

2. (10 points) Find an implicit general solution for $\frac{dy}{dx} = \frac{\cos^2(x)}{ye^y}$.

3. (5 points) Solve $\frac{dy}{dx} = 3y + 9$ if $y(\ln(2)) = 1$ using algebra and inspection, but not using separation of variables or the linear equation method.

4. (10 points) Match equations $y' = x+y$, $y' = x-y$, $y' = y+1$ to the slope fields. Justify your answers.

a)  b)  c) 

5. Are the following differential equations linear, separable, both, or neither? Circle the correct response. No work is required.

(a) (3 points) $x^2y' - x = 1 + y + xy$ Linear Separable Both Neither

(b) (3 points) $y' + 3x^2y = x^2$ Linear Separable Both Neither

(c) (3 points) $\sqrt{1-x^2}y' = xy - y^2$ Linear Separable Both Neither

(d) (3 points) $\sin(y)\cos(x)y' = \sin(x)y^2$ Linear Separable Both Neither

(e) (3 points) $xy' + yx^2 = \cos(x)$ Linear Separable Both Neither

6. Let $A = \sum_{n=1}^{\infty} \left(\frac{-3}{2}\right)^n$ and $B = \sum_{n=2}^{\infty} \left(\frac{3}{4}\right)^n$ for this problem.

   (a) (3 points) Which of the series A or B diverges and how do you know this?

   (b) (7 points) Calculate the convergent series.

7. (5 points) Calculate $L = \lim_{n\to\infty} \left(1 + \frac{2}{n}\right)^n$ or show that it diverges. Show complete work defending your answer.

8. (10 points) Use the comparison test or limit comparison test to determine if $S = \sum_{n=2}^{\infty} \frac{n}{\sqrt{n^5 - 1}}$ converges or diverges. Defend your answer by listing the series you are using for comparison, the test you are using, and by verifying conditions for that test.

9. (10 points) The following series diverge. Can the divergence test be used to show this? Explain why the test does or does not apply.

   (a) $\displaystyle\sum_{n=2}^{\infty} \frac{\ln(n)}{n}$

   (b) $\displaystyle\sum_{k=1}^{\infty} \cos\left(\frac{1}{k}\right)$

10. (15 points) Determine if the following series converge absolutely, converge conditionally, or diverge. Defend your answer as discussed in lecture.

    (a) $S = \displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}}$

    (b) $S = \displaystyle\sum_{n=1}^{\infty} \frac{\cos(n\pi)}{\sqrt{n^3}}$