Final 100 points Math 200 Name:_______________________________________

Circle or box, and simplify your final answers. Show work or some other defense of your answers.

1)Use Stokes' Theorem **once** to find I= $\iint_S \left(\nabla\times\vec{F}\right)\cdot d\vec{S}$ if $\vec{F}=\langle -y,x,z\rangle$ and S is the part of the sphere $\rho=2$ oriented **up** with $z\geq\sqrt{3}$. (5 points)

2) Find the flux of $\vec{F}=y\hat{\mathbf{j}}$ through the piece of $z=x^2+y^2$ that lies in the first octant inside the cylinder r = 1 and oriented up. (5 points)

3) Let $W(s,t)=F(u(s,t),v(s,t))$, where F, u, and v are differentiable with outputs given below. Find $W_t(1,0)$.

(5 points)

$u(1,0)=2$ $v(1,0)=3$

$u_s(1,0)=-2$ $v_s(1,0)=5$

$u_t(1,0)=6$ $v_t(1,0)=4$

$F_u(1,0)=5$ $F_v(1,0)=7$

$F_u(2,3)=-1$ $F_v(2,3)=10$

4) Use $u = y - x^3$ and $v = x + y$ to set up an integral in terms of u and v that equals I = $\iint_R \frac{3x^2+1}{x+y}\,dA$ if R is bounded by $y - x^3 = 1$, $y - x^3 = 3$, $x + y = 1$, and $x + y = 2$. **DO NOT EVALUATE**. (5 points)

5) Use the second partials test to find and classify all the critical points with $0 < x \le \pi$ for $f(x,y) = y^2 - \sin^2(x)$. (10 points)

6) Use spherical coordinates to **convert** I = $\int_0^{2\pi}\int_0^2\int_0^{\sqrt{4-r^2}} z \cdot r\,dz\,dr\,d\theta \;-\; \int_0^{2\pi}\int_0^1\int_0^{\sqrt{1-r^2}} z \cdot r\,dz\,dr\,d\theta$ to one triple integral that is ready to integrate, **BUT DO NOT EVALUATE**. (5 points)

7) Find the mass of the part of the the cylinder $r=2$ that lies between z = 0 and the plane $x+y+z=4$ if the density of the surface is $\delta(x,y,z)=\left(x^2+y^2\right)$ grams per square cm. (10 points)

8A) Find the volume of the solid E bounded by $z=5-x^2-y^2$ and z = 1. (5 points)

8B) Use the divergence theorem to find I = $\iint_{Bd(E)} \vec{F}\cdot d\vec{S}$ if $\vec{F}=\langle -y,x,z\rangle$ and E is the region in 8A. (3 points)

8C) Set up, **BUT DO NOT EVALUATE**, a triple integral equal to $\overline{z}$, the z – coordinate of the center of mass of E, if the density of E is 10 grams per cubic centimeter and E is the region in 8A. (2 points)

9A) Let $g(x,y,z) = x^2 - y^2 + z^2$. Find the equation of the tangent plane Ax + By + Cz = D to the level surface g(x,y,z) = 1 at the point P = (1, 2, 2). (5 points)

9B) Find the rate of change of $g(x,y,z) = x^2 - y^2 + z^2$ at the point P = (1, 2, 2) in the direction $\langle 3,0,4 \rangle$. (3 points)

9C) Use differentials to estimate g(1.2, 2.1, 1.7). From 9A we know g(1, 2, 2) = 1. (2 points)

10) Let $f(x,y) = x^2 + y^2$ be subjected to the constraint $4x^2 + y^2 = 4$. Use a Lagrange multiplier to find the global maximum and global minimum of f(x, y) on the constraint. Sketch a diagram showing the constraint and a few appropriate level curves of f(x, y). (10 points)

11A) **Fill in the blanks**: $\vec{F}$ is a gradient field if the domain of $\vec{F}$ is________________________ and the curl of $\vec{F}$ is________________________. (2 points)

11B) $\vec{F} = \langle e^y, xe^y + e^z, ye^z \rangle$ is a gradient field. Find a potential for $\vec{F}$. Use a method to find it and show it in an organized way. (6 points)

11C) Find the work done by $\vec{F} = \langle e^y, xe^y + e^z, ye^z \rangle$ on a particle as it moves along an arbitrary piecewise smooth path from (0, 2, 0) to (4, 0, 3). Hint: FTCLI. (2 points)

12) Evaluate I = $\int_C \sqrt{1+x^3}\,dx + 2xy\,dy$ if C is the piecewise linear path from (0, 0) to (1, 3) to (1, 0) to (0, 0). Hint: Use Green's Theorem. (5 points)

13) Evaluate I = $\int_C 3y\,dx + \frac{1}{2}\,dy$ if $C = C_1 + C_2$ where C_1 is the line segment from (0, 3) to (1, 1) and C_2 is the curve from (1, 1) to (3, 9) on $y = x^2$. (10 points)