Math 200
Homework #1 (24 problems)

Homework is graded for completeness and presentation; only the CAS problem is graded for correctness. Please make a margin on the left and put your circled problem numbers **in order** to the left of the margin. Leave space between each problem. Leave a margin on top so that when the papers are stapled the problem number in the upper left corner is visible, or put the problem number in the center of the page. Provide context for each problem. One-word answers rarely earn credit. Each problem is worth at least one point, even if there are 100 problems and only 25 points to award. The answer for the CAS problem may be placed before problem 1 instead of last if you wish. Number the problems from 1 to 26. You may leave off the page numbers, sections, and book problem #.

Find the sum, dot product, and cross product (in the given order) for each pair of vectors.

1) $\langle 1,1,1\rangle$, $\langle -3,1,2\rangle$ 2) $\langle 5,-1,3\rangle$, $\langle -10,2,-6\rangle$ 3) $\hat{\mathbf{i}}+2\hat{\mathbf{j}}+2\hat{\mathbf{k}}$, $\langle 2,1,-2\rangle$ 4) $\hat{\mathbf{i}}+\hat{\mathbf{k}}$, $\hat{\mathbf{i}}+\hat{\mathbf{j}}$

Use the pairs of vectors in #1-4 to answer # 5 - 9.

5) What is the work done by a constant force field represented by the first vector on a particle with displacement equal to the second vector? Assume the unit is J = Joules.
6) Which pairs are parallel vectors? Which pairs are perpendicular vectors?
7) What is the area of the parallelogram and triangle spanned by the vectors in #1?
8) Find the fluxes of the constant force field $\vec{F}(x,y,z)=\langle 1,1,1\rangle$ through the parallelograms spanned by the pairs of vectors in numbers 1 to 4. (units = g/sec)
9) What are the volumes of the boxes spanned by $\langle 1,1,1\rangle$ and the pairs of vectors in #1 – 4?

Let $\mathbf{v}=\langle -1,2,-2\rangle$ and $\mathbf{w}=3\hat{\mathbf{i}}-5\hat{\mathbf{j}}+4\hat{\mathbf{k}}$ for # 10 - 13.

10) Find $\|\mathbf{v}\|, \|\mathbf{w}\|$, and $\|\mathbf{v}+\mathbf{w}\|$. Verify that the sum of any two of these is larger than the third.
11) Find $\mathbf{v}_{\mathbf{w}\|}$, $\mathbf{v}_{\mathbf{w}\perp}$, $\mathbf{w}_{\mathbf{v}\|}$, and $\mathbf{w}_{\mathbf{v}\perp}$.
12) Find the cosine of the angle between **v** and **w**.
13) Find the equation of the plane through $(3,-1,2)$ that is parallel to **v** and **w**.

Let P be the plane x + 4y +7z = 3 for # 15 – 17.

14) Find a vector perpendicular to P.
15) Find two independent vectors that are parallel to P. ("Independent" means the two vectors are **not** parallel.)
16) Find the equation of the plane parallel to P passing through (2, 1, 2).
17) Find a position function for the line perpendicular to P that passes through (3, 0, 0).

18) Find a position function for the line passing through Q = $(-1,2,0)$ and R = $(2,-4,2)$ oriented from Q to R.

#19 – 23: Sketch the following surfaces using the positive orientation. Label the axes.

19) $x = -y^2 - z^2$ 20) $y = z^2 - x^2$ 21) $z = -\sqrt{y^2 + x^2}$ 22) $x^2 + z^2 = 4$

23) x + 3y + 5z = 15 in the first octant. The first octant is the region where x, y, and z are all positive.

24) CAS Problem (3 points): Use **MatLab** to solve the following. Use a live script and turn in a pdf copy.

Let $\vec{a} = \langle 4,7,-1 \rangle$, $\vec{b} = \langle 3,8,-2 \rangle$ and the point $C = (5,3,6)$.

a) Find $\vec{a}_{\vec{b}\parallel}$ and $\vec{a}_{\vec{b}\perp}$.

b) Compute the area of the parallelogram spanned by $\vec{a}$ and $\vec{b}$.

c) Compute the distance from the point C to the plane $2x - 5y + 7z = 8$.

Brief Answers:

1 - 4) Sums: $\langle -2,2,3 \rangle$, $\langle -5,1,-3 \rangle$, $\langle 3,3,0 \rangle$, $\langle 2,1,1 \rangle$ Dot Products: 0, - 70, 0, 1

Cross products: $\langle 1,-5,4 \rangle$, $\langle 0,0,0 \rangle$, $\langle -6,6,-3 \rangle$, $\langle -1,1,1 \rangle$

5) Work: 0 J, - 70 J, 0 J, 1J

6) The pairs in #1 and #3 are perpendicular. The pair in #2 is parallel. How do vector products indicate this?

7) $\sqrt{42}$ and $\frac{\sqrt{42}}{2}$ 8) Fluxes: 0 g/sec, 0 g/sec, - 3 g/sec, 1 g/sec 9) Volumes: 0, 0, 3, 1.

10) 3, $5\sqrt{2}$, $\sqrt{17}$ 11) $\mathbf{v}_{\mathbf{w}\parallel} = -.42\mathbf{w}$, $\mathbf{v}_{\mathbf{w}\perp} = \langle .26,-.1,-.32 \rangle$, $\mathbf{w}_{\mathbf{v}\parallel} = \frac{-7}{3}\mathbf{v}$, and $\mathbf{w}_{\mathbf{v}\perp} = \frac{\langle 2,-1,-2 \rangle}{3}$.

12) $\frac{-21}{15\sqrt{2}}$ 13) 2x + 2y + z = 6 14) $\langle 1,4,7 \rangle$

15) Any two independent vectors perpendicular to $\langle 1,4,7 \rangle$. e.g. $\langle 7,0,-1 \rangle$ and $\langle 4,-1,0 \rangle$.

16) x + 4y + 7z = 20 17) $\vec{p}(t) = \langle 3+t, 4t, 7t \rangle$ 18) $\vec{p}(t) = \langle -1+3t, 2-6t, 2t \rangle$

19 - 23) Sketches should be of (19) a paraboloid opening about the negative x – axis; (20) a saddle with the y-axis pointing up from the dip in the saddle; (21) a cone opening about the negative z – axis; (22) a cylinder about the y – axis; and (23) a plane over the first octant with intercepts of 15, 5, and 3.