Math 200
Homework #2: (31 problems)

Homework is graded for completeness and presentation; only the CAS problem is graded for correctness. Please make a margin on the left and put your circled problem numbers **in order** to the left of the margin. Leave space between each problem. Leave a margin on top so that when the papers are stapled the problem number in the upper left corner is visible, or put the problem number in the center of the page. Provide context for each problem. One-word answers rarely earn credit. Each problem is worth at least one point, even if there are 100 problems and only 25 points to award. The answer for the CAS problem may be placed before problem 1 instead of last if you wish. Number the problems from 1 to 26. You may leave off the page numbers, sections, and book problem #.

1) Convert $\left(2,\frac{\pi}{3},-8\right)_C$ to rectangular and spherical coordinates.

2) Convert $\left(5,\frac{3\pi}{4},\frac{\pi}{4}\right)_S$ to rectangular and cylindrical coordinates.

3) Convert $(0,-1,1)$ to cylindrical and spherical coordinates.

In #4 and 5 identify (by converting to rectangular coordinates) and then sketch the given surface.

4) $z=4-r^2$ 5) $\rho=2\sin\theta\sin\phi$

In # 6 and 7 , sketch the solid described by the given inequalities.

6) $\rho\le 2$, $\rho\le\csc(\phi)$ 7) $-\frac{\pi}{2}\le\theta\le 0$; $r\le z\le 2$

Sketch directed traces of $\boldsymbol{\alpha}(t)$ on sketched surfaces and write the equation of the surface.

8) $\boldsymbol{\alpha}(t)=(2\cos(t),2\sin(t),4t)$ 9) $\boldsymbol{\alpha}(t)=(2t\cos(t),2t\sin(t),2t)$ 10) $\boldsymbol{\alpha}(t)=(2t\cos(t),2t\sin(t),4t^2)$

Find a parameterization for the following directed curves C in terms of time t seconds.

11) C is the circle in z = 2 with radius 5, center (0, 1, 2), and directed clockwise with respect to the z – axis starting at the point (5, 1, 2) when t = 0. Furthermore, if t is time in seconds, C rotates four times each second and moves for five seconds.

12) C moves from (1, 2, 3) to (4, 6, 0) on a straight line in five seconds. Use t for the time parameter. Also give the symmetric form for the line that passes through (1, 2, 3) and (4, 6, 0).

13) C moves from $(-8,4)$ to $(27,9)$ on the path $x^2=y^3$.

Let $\vec{\alpha}(t)=\left\langle\sqrt{4-t^2},\frac{t}{\sin(t)},\ln(t+1)\right\rangle$**.**

14) Find the domain of $\vec{\alpha}(t)$**.** 15) Find $\lim_{t\to 0}\vec{\alpha}(t)$.

16) Two particles travel along the space curves $\vec{r}(t)=\langle 0,t,t^2\rangle$ and $\vec{\alpha}(t)=(1+2t)\hat{\mathbf{j}}+(1+4t)\hat{\mathbf{k}}$. Will the particles collide? Do the paths cross? Defend your answers.

17) Evaluate $\int_{-\pi}^{\pi} \cos^2(t)\hat{\mathbf{i}} + \sqrt{t^2+1}\sin(t)\hat{\mathbf{j}} - \sin(3t)\sin(t)\hat{\mathbf{k}}\, dt$

For #18 to 21 let $\vec{\alpha}(t) = \langle t^2, -4t, -t^2 \rangle$.

18) Find $\|\vec{\alpha}(t)\|$, $\vec{v}(t)$, $v(t)$, $\vec{a}(t)$.

19) Find the cosine of the angle between $\vec{a}(t)$ and $\vec{v}(t)$. For what values of t is $\vec{a}(t)$ perpendicular to $\vec{v}(t)$? When is it parallel to $\vec{v}(t)$?

20) Find the definite integral that is equal to the arc length from t = 0 to t = 2.

21) Find $\vec{v}(t) \times \vec{a}(t)$, the equation of the osculating plane at time t, and the curvature at time t.

22) Let $\vec{r}(t) = \cosh(t)\hat{i} + \sinh(t)\hat{j} + t\hat{k}$. Find the arc length from t = 0 to t = 2, and find the curvature at time t.

23) Let $\vec{p}(t) = t\cos(t)\hat{i} + t\sin(t)\hat{j} + \hat{k}$. Sketch the curve traced from t = 0 to t = 2, find an integral equal to its arc length, and find the curvature at time t.

24) Let $\vec{r}(t) = \langle x(t), y(t) \rangle$ and $\vec{\alpha}(t) = \langle a(t), b(t) \rangle$. Prove that $\frac{d}{dt}(\vec{r}(t) \cdot \vec{\alpha}(t)) = \frac{d\vec{r}(t)}{dt} \cdot \vec{\alpha}(t) + \vec{r}(t) \cdot \frac{d\vec{\alpha}(t)}{dt}$.

25) If the path $\vec{r}(t)$ at some point P has $\vec{a} = \langle 3,1,1 \rangle$ and $\vec{v} = \langle 2,2,1 \rangle$, find $\hat{T}$ and $\hat{N}$ at P.

26) Find the equation of the plane that contains the point P = $(1,1,0)$ and the line $\frac{x-2}{3} = \frac{y+1}{2} = \frac{z-3}{4}$.
Write your final answer in standard form.

For #27 and 28, graph at least six level curves and then draw the graph of the function.

27) $f(x,y) = \sqrt{x^2 + y^2}$

28) $g(x,y) = x^2 - y^2$

For #29 and 30 let $f(x,y,z) = e^{\sqrt{z - x^2 - y^2}}$.

29) Evaluate $f(2,-1,6)$

30) Find and sketch the domain of f(x, y, z).

31) CAS Problem (3 points): Use **MatLab** to solve the following. Use a live script and turn in a pdf copy.

Draw the trace of $\vec{\alpha}(t) = \langle 3\cos(t), 3\sin(t), 9\cos(2t) \rangle$ and the saddle it lies on in the same coordinate system. Use **fmesh** for the saddle so you can see the curve and notice the curve is periodic, so you only need to draw one period.

Brief Answers:

1) $\left(1,\sqrt{3},-8\right)$ and $\left(2\sqrt{17},\frac{\pi}{3},\cos^{-1}\left(\frac{-4}{\sqrt{17}}\right)\right)_S$ 2) $\left(\frac{-5}{2},\frac{5}{2},\frac{5}{\sqrt{2}}\right)$ and $\left(\frac{5}{\sqrt{2}},\frac{3\pi}{4},\frac{5}{\sqrt{2}}\right)_C$

3) $\left(1,\frac{3\pi}{2},1\right)_C$ and $\left(\sqrt{2},\frac{3\pi}{2},\frac{\pi}{4}\right)_S$ 4) A paraboloid 4 units high opening down.

5) $x^2+(y-1)^2+z^2=1$, a sphere of radius 1 translated 1 unit right along the y-axis.

6) A solid cylinder of radius 1 with spherical caps from a sphere of radius 2.

7) A quarter of a cone 2 units high above the fourth quadrant of the xy – plane.

#8 – 10: the curve should wind counterclockwise around on the
8) cylinder $x^2+y^2=4$,   9) double cone $x^2+y^2=z^2$, and 10) paraboloid $x^2+y^2=z$.

11) $\boldsymbol{\alpha}(t)=(5\cos(8\pi t),1-5\sin(8\pi t),2)$, $0\le t\le 5$.

12) $\boldsymbol{\alpha}(t)=(1+\frac{3t}{5},2+\frac{4t}{5},3-\frac{3t}{5})$, $0\le t\le 5$. Symmetric form for the line is $\frac{x-1}{3}=\frac{y-2}{4}=\frac{3-z}{3}$.

13) $\boldsymbol{\alpha}(x)=(x,x^{2/3})$, $-8\le x\le 27$ 14) Domain = $(-1,0)\cup(0,2]$ 15) $\langle 2,1,0\rangle$

16) No Collision; the paths cross at (0, 1, 1) at different times.   17) $\langle \pi,0,0\rangle$

18) $\|\vec{\alpha}(t)\|=\sqrt{2t^4+16t^2}$; $\vec{v}(t)=\langle 2t,-4,-2t\rangle$; $v(t)=2\sqrt{2t^2+4}$; $\vec{a}(t)=\langle 2,0,-2\rangle$

19) $\cos\theta=\frac{t}{\sqrt{t^2+2}}$; perpendicular if t = 0, never parallel.   20) $2\sqrt{2}\int_0^2\sqrt{t^2+2}\,dt$

21) $\langle 8,0,8\rangle$;   x + z = 0; $\frac{\sqrt{2}}{\left(2t^2+4\right)^{3/2}}$ 22) $\sqrt{2}\sinh(2)$; $\frac{1}{2\cosh^2(t)}$

23) Piece of a spiral in z = 1; $\int_0^2\sqrt{t^2+1}\,dt$; $\frac{t^2+2}{(t^2+1)^{3/2}}$ 25) $\frac{\langle 2,2,1\rangle}{3}$; $\frac{\langle 1,-1,0\rangle}{\sqrt{2}}$

26) $14x-5y-8z=9$

27) The level curves are circles with radius k $\ge$ 0. The graph is the upper cone.

28) The level curves are hyperbolas for k $\neq$ 0 and the lines $y=\pm x$ if k = 0. The graph is a saddle.

29) e   30) $z\ge x^2+y^2$. The graph is a paraboloid and all points inside it.