Math 200

Homework #3: (28 problems)

Answers to every homework question are given so your homework grade will depend only on completeness and presentation. I should be able to see at a glance that you have done all of the problems with sufficient work. Write your problems in order and write the circled problem number to the left of the margin; draw the margin if necessary. If you run out of room in the middle of a problem, write a note directing me to the end of the assignment and put the remainder of your work there. Leave a margin at the top of each side of your stapled papers so that the problem numbers and all of your work are visible.

#1 – 7: evaluate the limit or prove that it does not exist.

1) $\lim_{(x,y)\to(0,0)} \dfrac{x^2+y^2}{\sqrt{x^2+y^2+1}-1}$

2) $\lim_{(x,y)\to(0,0)} \dfrac{xy+xy^2}{x^2+y^2}$

3) $\lim_{(x,y)\to(0,0)} (2x+y)e^{-x+y}$

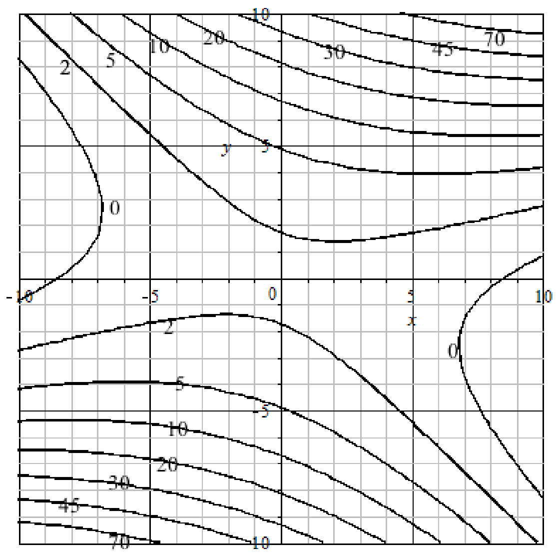
4) $\lim_{(x,y)\to(0,2)} \dfrac{\left(e^x-1\right)\left(e^y-1\right)}{x}$

5) $\lim_{(x,y)\to(0,0)} \dfrac{x^3y^2+x^2y^3}{x^4+y^4}$

6) $\lim_{(x,y)\to(0,0)} \dfrac{xy}{\sqrt{x^2+y^2}}$

7) $\lim_{(x,y)\to(3,5)} \dfrac{(x-3)^2+2(y-5)^2}{(x-3)^2+(y-5)^2}$

8) Use the contour diagram to estimate $\dfrac{\partial f}{\partial x}$ and $\dfrac{\partial f}{\partial y}$ at P = (0, 6).



For #9 and 10 compute all of the first and second partial derivatives, find the gradient, and then evaluate the gradient at (1, 1).

9) $f(x,y)=e^{-y}\cos(\pi x)$

10) $g(m,p)=m\ln(m^2p)$

For #11 and 12 compute the curl and divergence; then evaluate these at (1, 1, 1).

11) $\vec{w}(x,y,z)=\langle xy, yz, zx\rangle$

12) $\vec{F}(x,y,z)=\dfrac{\langle x,y,z\rangle}{x^2+y^2+z^2}$

13) Use implicit differentiation to find $\dfrac{\partial z}{\partial x}$ and $\dfrac{\partial z}{\partial y}$ if $x^2+y^2+z^2=3xyz$.

For #14 and 15, sketch the given vector field and guess if the work done by this field on a particle moving once around the unit circle counterclockwise is positive, negative, or 0. Then use a line integral to find the exact work done.

14) $\vec{F}(x,y)=y\hat{\mathbf{i}}+\hat{\mathbf{j}}$

15) $\vec{G}(x,y)=\dfrac{\langle -y,x\rangle}{\sqrt{x^2+y^2}}$

For #16 – 23, evaluate the line integral.

16) $\int_C y^3 ds$, C: $x = t^3$, $y = t$, $0 \le t \le 2$

17) $\int_C xy^4 ds$, C: $x^2 + y^2 = 4$, $x \ge 0$

18) $\int_C xyz ds$, C: $\vec{r}(t) = \langle 2\sin t, t, -2\cos t \rangle$, $0 \le t \le \pi$

19) $\int_C xe^{yz} ds$, C: line segment from (0, 0 0) to (1, 2, 2).

20) $\int_C \vec{F} \cdot d\vec{s}$, $\vec{F}(x, y) = xy\hat{\mathbf{i}} + 3y^2\hat{\mathbf{j}}$, and C: $\vec{r}(t) = t^4\hat{\mathbf{i}} + t^3\hat{\mathbf{j}}$ with $0 \le t \le 1$.

21) $\int_C \vec{F} \cdot d\vec{s}$, $\vec{F}(x, y, z) = \langle \sin x, \cos y, xz \rangle$, and C: $\vec{r}(t) = \langle t^3, -t^2, 5t \rangle$ with $0 \le t \le 1$.

22) $\int_C \sin x \, dx + \cos y \, dy$, C is the line segment from (0, 0) to (3, 2).

23) $\int_C (2x + yz) \, dx + 2x \, dy + xyz \, dz$, C is the piecewise linear curve from (0, 1, 4) to (1, 1, 4) to (1, 2, 4) to (1, 2, 8). "piecewise linear" means "made of line segments."

24) Calculate the total mass of a wire $\vec{\alpha}(t) = \langle \cos t, \sin t, t^2 \rangle$ cm for $0 \le t \le 2$ if the density is $\delta(x, y, z) = \sqrt{z}$ grams/cm.

25) Calculate the total mass of a wire on the line segment from (1, 1, 1) to (1, 4, 5) cm with density $\delta(x, y, z) = x + y + z$ grams/cm.

26) Calculate the work done by $\vec{F}(x, y) = \langle x + 2y, x - y \rangle$ on a particle that moves from (0, 0) to (1, 1) along the path $x = y^2$.

27) Calculate the work done by $\vec{F}(x, y, z) = x\hat{\mathbf{i}} + y\hat{\mathbf{j}} + z\hat{\mathbf{k}}$ on a particle that moves along $\vec{p}(t) = \cos(t)\hat{\mathbf{i}} + \sin(t)\hat{\mathbf{j}} + t\hat{\mathbf{k}}$ for $0 \le t \le 3\pi$.

28) CAS Problem (3 points): Use **MatLab** to solve the following. Use a live script and turn in a pdf copy.

A) Draw a contour plot for the function $f(x, y) = (x^2 + y^2)^2 - x^2 + y^2$ drawing and labeling the contour curves for which $f(x, y) =$ -0.2, 0, 2, 5, 10, 20, and 30 using the domain $[-2, 2] \times [-2, 2]$.

B) Draw the graph of $f(x, y) = (x^2 + y^2)^2 - x^2 + y^2$ using the same domain. Notice no matter how you rotate the graph, you cannot see the minimum values.

C) Draw a graph for $f(x, y) = (x^2 + y^2)^2 - x^2 + y^2$ for which you *can* see the minimum values. You may restrict the domain or use the command **min(*)** to set all positive outputs to 0.

Brief answers.

1) 2 2) DNE 3) 0 4) e^2-1 Hint: product rule.

5) 0 Hint: $\left(x^2-y^2\right)^2 \geq 0 \Rightarrow x^2y^2 \leq x^4+y^4$ 6) 0 7) DNE

8) $\frac{\partial f}{\partial x}(0,6) \approx 1.25, \ \frac{\partial f}{\partial y}(0,6) \approx 2.5.$

9) $f_x(x,y) = -\pi e^{-y}\sin(\pi x), \ f_y(x,y) = -e^{-y}\cos(\pi x), \ \nabla f(x,y) = \left\langle -\pi e^{-y}\sin(\pi x), -e^{-y}\cos(\pi x)\right\rangle$
$f_{xx}(x,y) = -\pi^2 e^{-y}\cos(\pi x), \ f_{yy}(x,y) = e^{-y}\cos(\pi x), \ f_{xy}(x,y) = f_{yx}(x,y) = \pi e^{-y}\sin(\pi x),$
$\nabla f(1,1) = \left\langle 0, e^{-1}\right\rangle$

10) $g_m(m,p) = \ln(m^2p)+2, \ g_p(m,p) = \frac{m}{p}, \ \nabla g(m,p) = \left\langle \ln(m^2p)+2, \frac{m}{p}\right\rangle$
$g_{mm}(m,p) = \frac{2}{m}, \ g_{pp}(m,p) = \frac{-m}{p^2}, \ g_{mp}(m,p) = g_{pm}(m,p) = \frac{1}{p}, \ \nabla g(1,1) = \left\langle 2,1\right\rangle$

11) $\nabla\cdot\vec{w}(x,y,z) = x+y+z$; $\nabla\cdot\vec{w}(1,1,1) = 3$; $\nabla\times\vec{w}(x,y,z) = \left\langle -y,-z,-x\right\rangle$; $\nabla\times\vec{w}(1,1,1) = \left\langle -1,-1,-1\right\rangle$

12) $\nabla\cdot\vec{F}(x,y,z) = \frac{1}{x^2+y^2+z^2}$; $\nabla\cdot\vec{F}(1,1,1) = \frac{1}{3}$; $\nabla\times\vec{F}(x,y,z) = \left\langle 0,0,0\right\rangle$; $\nabla\times\vec{F}(1,1,1) = \left\langle 0,0,0\right\rangle$

13) $z_x = \frac{2x-3yz}{3xy-2z}$ and $z_y = \frac{2y-3xz}{3xy-2z}$

14) The arrows are longer when starting further from the x – axis, and they point along parabolic curves. Work = $-\pi$. The arrows have tangential components mostly pointing clockwise.

15) The arrows are all one unit long and point along circles, so work should be positive. Work = 2π.

16) $\frac{145^{3/2}-1}{54}$ 17) $\frac{128}{5}$ 18) $\sqrt{5}\pi$ 19) $\frac{3}{8}\left[e^4-1\right]$

20) $\frac{15}{11}$ 21) $6-\cos(1)-\sin(1)$ 22) $1-\cos(3)+\sin(2)$

23) 55 24) $\frac{17^{3/2}-1}{12}$ grams 25) 32.5 grams

26) 5/3 joule 27) $\frac{9\pi^2}{2}$ joules