Math 200

Homework #4: (26 problems)

Answers to every homework question are given so your homework grade will depend only on completeness and presentation. I should be able to see at a glance that you have done all of the problems with sufficient work. Write your problems in order and write the circled problem number to the left of the margin; draw the margin if necessary. If you run out of room in the middle of a problem, write a note directing me to the end of the assignment and put the remainder of your work there. Leave a margin at the top of each side of your stapled papers so that the problem numbers and all of your work are visible.

For #1 and #2 find an equation of the tangent plane for the given surface at the given point.

1) $z = \sqrt{xy}$; $(1,4,2)$ 2) $z = y\cos(x-y)$; $\left(\pi, \frac{\pi}{2}, 0\right)$

3) Find a linear approximation for $f(x,y) = \frac{x}{\sqrt{x+y}}$ at (3, 1). use the linear approximation to estimate $f(3.1, 0.9)$.

For #4 and #5 find the differential of the given function.

4) $z = x^3 \ln(y^2)$ 5) $R = \alpha\beta^2 \cos\gamma$

6) The period of a simple pendulum is $T = 2\pi\sqrt{\frac{l}{g}}$ where l is the length and g is the gravitational constant. We compute T by taking π = 3 (|error| < 0.2), l = 40 cm (|error| < 0.1), and g = 1000 $cm/\sec^2$ (|error| < 20). Use differentials to estimate the maximum possible error in our calculation of T.

7) Three positive numbers, each less than or equal to 10, are rounded to the first decimal place (e.g. 1.0532 rounded to 1.1, 1.0499 to 1.0) and then multiplied together. Use differentials to estimate the maximum difference between the product of rounded numbers and the product of unrounded numbers.

8) The body mass index I of a person with mass M and height H is $I = \frac{M}{H^2}$. A man with actual mass M = 84 kg and actual height H = 2 meters uses measuring devices that have a maximum error of 2 kg and 0.1 meter respectively. **Use differentials** to estimate the maximum possible difference between his actual body mass index and the body mass index calculated using the numbers from the measuring devices.

9) The surface area of a human body in square feet = $S = 0.1091 w^{0.425} h^{0.725}$, w = weight (lb's) and h = height (inches). There is a maximum error of 2% when measuring w and h. Use differentials to estimate the maximum percentage error in calculated S.

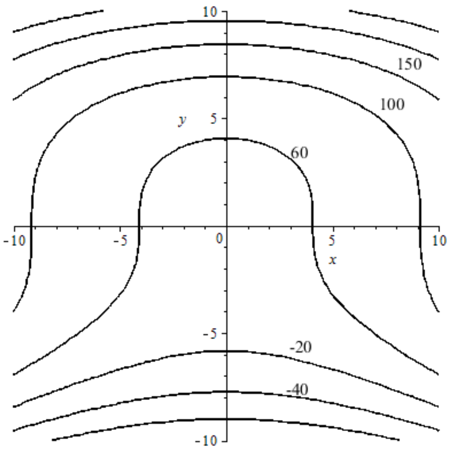
For #10 – 12 find the rate of change of the given function at P in the direction $\vec{v}$.

10) $f(x,y) = \sin(2x+3y)$, P = $(-6,4)$, and $\vec{v} = \langle\sqrt{3}, -1\rangle$.

11) $g(x,y,z) = xe^{2yz}$, P = $(3,0,2)$ and $\vec{v} = 2\hat{\mathbf{i}} - 2\hat{\mathbf{j}} + \hat{\mathbf{k}}$.

12) $h(x,y,z) = xe^y + ye^z + ze^x$, P = $(0,0,0)$, and $\vec{v} = \langle 5,1,-2\rangle$.

13) Sketch $\nabla f(0,-7)$ on the contour plot with correct length and direction. Then estimate $D_{-\hat{j}} f(0,5)$



14) Find the maximum and minimum rate of change of $g(x,y)=\frac{y^2}{x}$ at (2, 4) and give the direction in which it occurs. In what directions will the rate of change be 0 at this point?

15) Find the equation of the tangent plane to the surface $x^2-2y^2+z^2=2-yz$ at $(2,1,-1)$.

16) Find the equation of the tangent plane to the surface $z+1=xe^y\cos z$ at (1,0,0).

17) The temperature in a metal ball = T(x,y,z) > 0. T is inversely proportional to the distance from the center, (0, 0, 0). In what direction does T increase most rapidly at any point P = (x, y, z) in the ball? Defend your answer.

18) f(x,y) = xy. Find the tangent to the level curve f(x, y) = 6 at the point (3, 2). Sketch the level curve, the tangent line, and $\nabla f(3,2)$.

19) Find $\frac{\partial g}{\partial t}$ when w = 2 and t = 1, if $g=g(x(w,t),y(w,t))$, x(2, 1) = 3, $y=\frac{w}{t}$, $g_x(2,1)=4$, $g_x(3,2)=5$, $\frac{\partial x}{\partial t}=t+w$, and $g_y=yx^2$.

20) $z=\sqrt{1+x^2+y^2}$, $x=\ln t$, $y=\cos t$. Find $\frac{\partial z}{\partial t}$ using the chain rule.

21) Use the chain rule to find $\frac{\partial z}{\partial p}$ and $\frac{\partial z}{\partial t}$ if $z=x^2y^3$, $x=p\cos t$, and $y=p\sin t$.

22) Suppose $F=F(x,y)$, $G=G(t)$, and $h(x,y)=2F(x+y,y^2)+5G(xy)$. Furthermore, $F_x(3,1)=4$, $F_y(3,1)=5$, and $G'(2)=-4$. Find $h_y(2,1)$.

23) $u=x^2+yz$, $x=pr\cos\theta$, $y=pr\sin\theta$, $z=p+r$. Find $\frac{\partial u}{\partial p}$, $\frac{\partial u}{\partial r}$, $\frac{\partial u}{\partial \theta}$ when p = 2, r = 3, and $\theta=0$ using the chain rule. In what variable (p, r, or θ) will a small positive change result in the largest change in u? Why?

24) The radius of a right circular cone is increasing at a rate of 1.8 m/sec while the height is decreasing at 2.4 m/sec. At what rate is the volume of the cone changing when r = 1 m and h = 1.5 m? ($V=\frac{1}{3}\pi r^2 h$)

25) Find $\frac{\partial f}{\partial x}$ in terms of x, y, and the function g if $f(x,y)=\int_{x+y}^{xy} g(t)\,dt$. Assume g has all real numbers for its domain and g is continuous. (Review the second fundamental theorem of calculus.)

26) CAS Problem (3 points): Use **MatLab** to solve the following. Use a live script and turn in a pdf copy.

Superimpose the gradient field for $f(x,y)=(x^2+y^2)^2-x^2+y^2$ on the contour plot from homework three. Recall the level curves were $f(x,y)=$ -0.2, 0, 2, 5, 10, 20, and 30 and the domain was $[-2,2]\times[-2,2]$. Do not label the level curves this time. If done numerically, you will need to use a coarser mesh for the quiver function when drawing the gradient field than was used for the contour plot. You should also "normalize" the gradient field – make all the arrows uniform length – so that none overlap. Please notice how the vectors in the gradient field are perpendicular to the level curves.

Brief answers.

1) $4x+y-4z=0$ 2) $2\pi x-2\pi y+4z=\pi^2$

3) $f(x,y)\approx\frac{3}{2}+\frac{5}{16}(x-3)-\frac{3}{16}(y-1)$; $f(3.1,0.9)\approx 1.55$

4) $dz=3x^2\ln(y^2)\,dx + \frac{2x^3}{y}\,dy$ 5) $dR=\beta^2\cos\gamma\,d\alpha + 2\alpha\beta\cos\gamma\,dB - \alpha\beta^2\sin\gamma\,d\gamma$

6) 0.0935 7) 15 8) 2.6 9) 2.3% 10) $\sqrt{3}-1.5$

11) $\frac{-22}{3}$ 12) $\frac{4}{\sqrt{30}}$ 13) Should be about $10\hat{\mathbf{j}}$, and $D_{-\hat{\mathbf{j}}}f(0,5)\approx\frac{-40}{3}$.

14) Max = $4\sqrt{2}$ in direction $\langle -1,1\rangle$, Min = $-4\sqrt{2}$ in direction $\langle 1,-1\rangle$.
Zero in directions $\langle 1,1\rangle$ and $\langle -1,-1\rangle$.

15) $4x-5y-z=4$ 16) $x+y-z=1$

17) Increases most rapidly towards the origin because the gradient points in that direction.

18) Tangent line: 2x +3y = 12; draw a picture. 19) -21

20) $\frac{\partial z}{\partial t}=\frac{\ln t-t\sin t\cos t}{t\sqrt{1+(\ln t)^2+\cos^2 t}}$

21) $\frac{\partial z}{\partial p}=5p^4\cos^2 t\sin^3 t$ $\frac{\partial z}{\partial t}=-2p^5\cos t\sin^4 t+3p^5\cos^3 t\sin^2 t$ 22) -12

23) $u_p=36$, $u_r=24$, $u_\theta=30$; changing p produces the largest change because its rate of change is largest at 36.

24) $\pi\ \ m^3/\sec$. 25) $yg(xy)-g(x+y)$