Math 200
Homework #5: (26 problems)

Answers to every homework question are given so your homework grade will depend only on completeness and presentation. I should be able to see at a glance that you have done all of the problems with sufficient work. Write your problems in order and write the circled problem number to the left of the margin; draw the margin if necessary. If you run out of room in the middle of a problem, write a note directing me to the end of the assignment and put the remainder of your work there. Leave a margin at the top of each side of your stapled papers so that the problem numbers and all of your work are visible.

For #1 – 4 find a potential for the given gradient fields.

1) $\vec{F}(x,y) = e^x \sin y\hat{\mathbf{i}} + e^x \cos y\hat{\mathbf{j}}$

2) $\vec{G}(x,y) = \left\langle \ln y + 2xy^3, 3x^2y^2 + \dfrac{x}{y} \right\rangle$, y > 0.

3) $\vec{H}(x,y,z) = \left\langle x^{-1}z, y^{-1}z, \ln(xy) \right\rangle$, x, y > 0.

4) $\vec{K}(x,y,z) = \left(2xy + \sqrt{1+z^2}\right)\hat{\mathbf{i}} + \left(x^2 + \sin z\right)\hat{\mathbf{j}} + \left(y\cos z + \dfrac{xz}{\sqrt{1+z^2}}\right)\hat{\mathbf{k}}$

In #5 – 8 use the Fundamental Theorem of Calculus for Line Integrals (FTCLI) to evaluate the following line integrals. The vector fields are the same as those in #1 – 4.

5) $\int_C \vec{F} \cdot d\vec{s}$, C: $\vec{r}(t) = \left\langle 1+t, \dfrac{\pi}{t} \right\rangle$, $1 \le t \le 2$.

6) $\int_C \vec{G} \cdot d\vec{s}$, C: starts at (0, 10) and spirals about (0, 5) in the upper half plane 10 times before stopping at (1 , e).

7) $\int_C \vec{H} \cdot d\vec{s}$, C: wiggles from $(1,e,1)$ to $(e,1,-1)$ on a path with positive x and y.

8) $\int_C \vec{K} \cdot d\vec{s}$, C: $\vec{\alpha}(t) = t^2\hat{\mathbf{i}} + -2t\hat{\mathbf{j}}$, $0 \le t \le 1$.

In #9 and 10, evaluate using the FTCLI if possible; otherwise use our older methods to calculate the line integral.

9) $\int_C \tan y\, dx + x\sec^2 y\, dy$, C: $\vec{p}(t) = \left\langle t+1, \dfrac{\pi t}{4} \right\rangle$, $0 \le t \le 1$.

10) $\int_C \dfrac{-y}{x^2+y^2} dx + \dfrac{x}{x^2+y^2} dy$, C: $\vec{r}(t) = \left\langle \cos t, \sin t \right\rangle$, $0 \le t \le 2\pi$.

11) Find constants p and a so that $\vec{F} = \left\langle \frac{2x}{px^2+2y^2}, \frac{ay}{x^2+y^2} \right\rangle$ is conservative. Find a potential. Is the domain simply connected? Evaluate $\int_C \vec{F} \cdot d\vec{s}$, C: $\vec{r}(t) = \langle \cos t, \sin t \rangle$, $0 \le t \le 2\pi$.

12) Use a Riemann sum with m = n = 2 to estimate $\iint_R \sin(x+y)\, dA$, R = $[0,\pi]\times[0,\pi]$. Use the lower left corners for points.

13) Use symmetry to evaluate the following.

13A) $\iint_R x^3\, dA$, R = $[-4,4]\times[0,5]$.

13B) $\iint_R 2 + x^2 y\, dA$, R = $[0,1]\times[-1,1]$

#14 – 17: Calculate the double integral.

14) $\int_0^{\pi/6}\int_0^{\pi/3} x\sin(x+y)\, dydx$

15) $\int_0^{\pi/2}\int_0^{\cos\theta} e^{\sin\theta}\, drd\theta$

16) $\iint_D y^2 e^{xy} dA$; D is bounded by y = 0, y = 4, x = 0, and y = x.

17) $\iint_D y\, dA$; D is the triangular region with vertices at (0, 2), (1, 1), and (3, 2).

#18, 19: Evaluate by reversing the order of integration. Sketch the region of integration.

18) $\int_0^4\int_{\sqrt{x}}^2 \frac{1}{y^3+1}\, dydx$

19) $\int_0^1\int_{\arcsin y}^{\pi/2} \cos x\sqrt{1+\cos^2 x}\, dxdy$

20) Find the volume of the solid under $x+2y-z=0$ and above the region in the xy-plane bounded by $y=x$ and $y=x^4$.

#21 – 23: Evaluate the triple integral.

21) $\int_0^{\pi/2}\int_0^y\int_0^x \cos(x+y+z)\, dzdxdy$

22) $\iiint_E x^2 e^y\, dV$, where E is bounded by $z = 1-y^2$, z = 0, x = 1, and x = -1.

23) $\iiint_T x^2\, dV$, where T is the solid bounded by x + y + z = 1 and the coordinate planes.

24) Rewrite $\int_1^2\int_0^{\sqrt{4-z^2}}\int_0^{\sqrt{4-y^2-z^2}} f(x,y,z)\, dxdydz$ if the order of integration is changed to $dydzdx$.

25) Find the mass of the solid satisfying $x,y,z \ge 0$, $z^2 \le 4x$, and $x^2+y^2 \le 16$ if the density is $\delta(x,y,z) = xyz^3$ grams / cm^3 and the units of x, y, and z are cm.

26) CAS Problem (3 points): Use **MatLab** to solve the following. Use a live script and turn in a pdf copy.

A) Find the work done by the vector field $\vec{F}(x,y)=\left\langle \frac{-y}{2},\frac{x}{2}\right\rangle$ on a particle displaced over the curve C parameterized by $\vec{\alpha}(t)=\langle t-\sin(t),1-\cos(t)\rangle$ for $0\le t\le 2\pi$. Use MatLab commands for all substitutions and differentiations.

B) Use MatLab to calculate $\int_0^{\pi/2}\int_0^{y}\int_0^{x}\sin(x+y+z)\,dzdxdy$.

Brief answers.

1) $\phi=e^x\sin y$ 2) $\phi=x\ln y+x^2y^3$ 3) $\phi=z\ln(xy)$ 4) $\phi=x^2y+x\sqrt{1+z^2}+y\sin z$

5) e^3 6) $1+e^3$ 7) -2 8) -1 9) 2 10) 2π

11) $a=1$, p = 2, and $\phi=\ln\left(\sqrt{x^2+y^2}\right)$. The line integral is 0; show this with and without the FTCLI.

12) $\frac{\pi^2}{2}$ 13A) 0 13B) 4

14) $\frac{6\sqrt{3}-\pi-6}{12}$ 15) $e-1$ 16) $\frac{e^{16}-17}{2}$ 17) 2.5

18) $\frac{\ln(9)}{3}$ 19) $\frac{2\sqrt{2}-1}{3}$ 20) $\frac{7}{18}$ 21) $\frac{-1}{3}$ 22) $\frac{8}{3e}$

23) $\frac{1}{60}$ 24) $\int_0^{\sqrt{3}}\int_1^{\sqrt{4-x^2}}\int_0^{\sqrt{4-x^2-z^2}} f(x,y,z)\,dydzdx$ 25) $\frac{2^{11}}{3}$ grams.