# Math 200
## Homework #6: (25 problems)

Answers to every homework question are given so your homework grade will depend only on completeness and presentation. I should be able to see at a glance that you have done all of the problems with sufficient work. Write your problems in order and write the circled problem number to the left of the margin; draw the margin if necessary. If you run out of room in the middle of a problem, write a note directing me to the end of the assignment and put the remainder of your work there. Leave a margin at the top of each side of your stapled papers so that the problem numbers and all of your work are visible.

**#1 – 5: use polar coordinates to evaluate the integrals. Sketch each region of integration.**

1) $\iint_R x^2 + y^2 \, dA$; R: $4 \le x^2 + y^2 \le 16$.

2) $\iint_R \frac{y}{x^2 + y^2} \, dA$; R: $y \ge \frac{1}{2}$ and $x^2 + y^2 \le 1$.

3) $\int_1^2 \int_{-\sqrt{2x-x^2}}^{\sqrt{2x-x^2}} \tan^{-1}\left(\frac{y}{x}\right) dydx$.

4) $\iint_R x - y \, dA$; R: $x^2 + y^2 \le 1$, $x + y \ge 1$, and $y \le x$.

5) $\iint_R x \, dA$; R: inside r = 2 with x > 0, and outside $(x-1)^2 + y^2 = 1$.

**#6 – 9: use cylindrical coordinates to evaluate the integrals.**

6) $\iiint_E z \, dV$; E: bounded by $x^2 + y^2 = 1$, z = 0, and $z = 4 + x^2 + y^2$.

7) $\iiint_E y\sqrt{x^2 + y^2} \, dV$; E: $x^2 + y^2 \le z \le 8 - \left(x^2 + y^2\right)$ and $y \ge 0$.

8) $\iiint_E x^2 \, dV$; E: inside r = 1, above z = 0, and below $z^2 = 4r^2$.

9) $\int_{-2}^{2} \int_{-\sqrt{4-y^2}}^{\sqrt{4-y^2}} \int_{\sqrt{x^2+y^2}}^{2} xz \, dzdxdy$

10) Use cylindrical coordinates to calculate the volume of $\rho \le a$ with r < b removed from it. Of course, b < $a$.

**#11 – 13: use spherical coordinates to evaluate the integrals.**

11) $\iiint\limits_E z\,dV$; E: $1 \le x^2 + y^2 + z^2 \le 4$ in the first octant.

12) $\iiint\limits_E x^2\,dV$; E is bounded by z = 0, $z = \sqrt{1-r^2}$, and $z = \sqrt{4-r^2}$.

13) $\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \sqrt{x^2+y^2+z^2}\,e^{-(x^2+y^2+z^2)}\,dxdydz$.

14) Convert $\int_0^1\int_0^{\sqrt{1-x^2}}\int_{\sqrt{x^2+y^2}}^{\sqrt{2-x^2-y^2}} f(x,y,z)\,dzdydx$ to an integral that uses spherical coordinates.

15) Compute the center of mass of the plate in the xy-plane that occupies the rectangle $[0,2]\times[-1,1]$ if the density is $\delta(x,y) = xy^2$.

16) Find the centroid (the center of mass if the density is constant) of the solid tetrahedron bounded by the coordinate planes and $x + y + z = 1$. Hint: calculate one coordinate and then use symmetry.

17) Calculate $I_x$ for the disk $r \le 2$ (units: meters) with density $1\ \mathrm{kg/m^2}$. How much kinetic energy (unit: joules) is required to rotate the disk about the x – axis with an angular velocity of 10 rad/sec?

18) Find center of mass for the part of the solid $\rho \le 1$ for which $z \ge 0$ if the density is $2\rho$ grams/cm$^3$.

19) Find $I_0$ for the solid in #18.

**#20 – 22: Use Green's theorem to evaluate $\int\limits_C \vec{F}\cdot d\vec{s}$.**

20) $\int\limits_C xy^2dx + 2x^2y\,dy$; C is the triangle from (0, 0) to (2, 2) to (2, 4) to (0, 0).

21) $\int\limits_C \left(y + e^{\sqrt{x}}\right)dx + \left(2x + \cos y^2\right)dy$; C is the boundary of the region enclosed by the parabolas $y = x^2$ and $x = y^2$ oriented positively.

22) $\vec{F} = \left\langle e^x + x^2y, e^y - xy^2 \right\rangle$; C is r = 5 oriented clockwise.

23) When a circle with radius one rolls along the outside of r = 4, a fixed point P on it traces a curve with parametric equations $x = 5\cos t - \cos(5t)$; $y = 5\sin t - \sin(5t)$. Sketch a graph of the curve and then use Green's theorem to find the area inside it.

24) A lamina in the xy-plane with constant density k is bounded by a positively oriented curve C. Show that $I_y = \frac{k}{3}\oint_C x^3\, dy$.

25) CAS Problem (3 points): Use **MatLab** to solve the following. Use a live script and turn in a pdf copy.

A) Use a triple integral to find the exact volume of the region that lies inside the sphere $x^2 + y^2 + z^2 = 4$ and inside the cylinder $x^2 + y^2 = 2x$.

B) Use a double integral to find the exact value of $\iint_R \frac{1}{\sqrt{x^2 + y^2}}\, dA$ if is in the right half of the plane ($x \ge 0$) and inside the ellipse $x^2 + 4y^2 = 9$ and outside the circle $(x-1)^2 + y^2 = 1$. The exact answer will have an unfamiliar function in it, so estimate this exact answer to four digits.

**Brief answers:**

1) $120\pi$ 2) $\sqrt{3} - \frac{\pi}{3}$ 3) 0 4) $\frac{4\sqrt{2} - 5}{12}$ 5) $\frac{16}{3} - \pi$

6) $\frac{61\pi}{6}$ 7) $\frac{64}{3}$ 8) $\frac{2\pi}{5}$ 9) 0 10) $\frac{4\pi\left(a^2 - b^2\right)^{3/2}}{3}$

11) $\frac{15\pi}{16}$ 12) $\frac{62\pi}{15}$ 13) $2\pi$ 14) $\int_0^{\pi/2}\int_0^{\pi/4}\int_0^{\sqrt{2}} f(\rho,\theta,\phi)\,\rho^2 \sin\phi \, d\rho d\phi d\theta$

15) $\left(\frac{4}{3}, 0\right)$ 16) $\left(\frac{1}{4}, \frac{1}{4}, \frac{1}{4}\right)$ 17) $I_x = 4\pi$, K.E. = $200\pi$ Joules. 18) $\left(0, 0, \frac{2}{5}\right)$

19) $\frac{2\pi}{3}$ 20) 12 21) 1/3 22) $\frac{625\pi}{2}$ 23) $30\pi$