Math 200
Homework #7: (25 problems)

Answers to every homework question are given so your homework grade will depend only on completeness and presentation. I should be able to see at a glance that you have done all of the problems with sufficient work. Write your problems in order and write the circled problem number to the left of the margin; draw the margin if necessary. If you run out of room in the middle of a problem, write a note directing me to the end of the assignment and put the remainder of your work there. Leave a margin at the top of each side of your stapled papers so that the problem numbers and all of your work are visible.

1) Find the equation of the tangent plane for the surface S: $\vec{p}(u,v) = u^2\hat{\mathbf{i}} + 2u\sin v\hat{\mathbf{j}} + u\cos v\hat{\mathbf{k}}$ when u = 1 and v = 0.

2) Find the area of the part of $z = y^2 - x^2$ that lies between r = 1 and r = 2.

3) Evaluate $\iint_S \frac{x^2yz}{1+2x+3y}\,dS$, S: the part of z = 1 + 2x + 3y that lies above the rectangle [0,3] x [0, 2].

4) Evaluate $\iint_S \vec{F}\cdot d\vec{S}$ if $\vec{F}(x,y,z) = x\hat{\mathbf{i}} + y\hat{\mathbf{j}} + z^4\hat{\mathbf{k}}$ and S is the part of z = r that lies beneath z = 1 and is oriented down.

5) Evaluate $\iint_S yz\,dS$ if S: $\vec{p}(u,v) = \left\langle u^2, u\sin v, u\cos v\right\rangle$ for $0 \le u \le 1$ and $0 \le v \le \frac{\pi}{2}$.

6) Evaluate $\iint_S x^2z + y^2z\,dS$ if S is the part of $\rho = 2$ for which $z \ge 0$.

7) Find the mass of the part of $y^2 + z^2 = 1$ that lies between x = 0 and x = 3 **in the first octant** if the density is $\delta(x,y,z) = z + x^2y$ grams/cm^2.

8) Find the mass of the part of $z = 4 - x^2 - y^2$ for which $0 \le z \le 3$ if the density is $\delta(x,y,z) = \frac{x^2}{4-z}$ grams/cm^2.

9) Find the mass of the part of $z = x^3$ for which $0 \le x \le 1$ and $0 \le y \le 1$ if the density is $\delta(x,y,z) = z$ grams/cm^2.

10) Find the surface area of the part of the sphere $x^2 + y^2 + z^2 = 4z$ that lies inside $z = x^2 + y^2$. Hint: Translate first.

11) Evaluate $\iint_S z^2\hat{\mathbf{i}}\cdot d\vec{S}$ if S: $\vec{p}(u,v) = u\cos v\hat{\mathbf{i}} + u\sin v\hat{\mathbf{j}} + v\hat{\mathbf{k}}$ is oriented up and $0 \le u \le 1$, $0 \le v \le 2\pi$.

12) Evaluate $\iint_S \langle xze^y, -xze^y, z^2\rangle \cdot d\vec{S}$ if S is the part of x + y + z = 1 that lies in the first octant **oriented down**.

13) Find the flux of $\vec{F}(x,y,z) = x\hat{\mathbf{i}} - z\hat{\mathbf{j}} + y\hat{\mathbf{k}}$ through the part of $x^2 + y^2 + z^2 = 4$ that lies in the first octant oriented towards the origin.

14) Find the flux of $\vec{F}(x,y,z) = y\hat{\mathbf{j}} - z\hat{\mathbf{k}}$ through the part of $z = xe^y$ with $0 \le x \le 1$ and $0 \le y \le 1$, oriented up.

15) Find the flux of $\vec{F}(x,y,z) = \langle x^2, y^2, z^2\rangle$ through the boundary of the solid half cylinder $0 \le z \le \sqrt{1 - y^2}$, $0 \le x \le 2$, oriented out.

16) Find the center of mass of the part of the surface $x^2 + y^2 + z^2 = a^2$ with $z \ge 0$ if it has constant density.

17) Let $\vec{e}_\rho = \dfrac{\langle x, y, z\rangle}{\rho}$ where, as usual, $\rho = \sqrt{x^2 + y^2 + z^2}$. Find the flux of $\vec{F}(x,y,z) = \dfrac{\vec{e}_\rho}{\rho^2}$ out of the sphere $x^2 + y^2 + z^2 = R^2$. Does changing R change the flux?

18) Evaluate $\iint_R x - 3y \, dA$ if R is the triangular region with vertices (0, 0), (2, 1), (1, 2) by making the substitution x = 2u + v, y = u + 2v.

19) Evaluate $\iint_R xy \, dA$ using a substitution if R is bounded by y = x, y = 3x, xy = 1, and xy = 3.

20) Evaluate $\iint_R \dfrac{x - 2y}{3x - y} \, dA$ using a substitution if R is bounded by x – 2y = 0, x – 2y = 4, 3x – y = 1, and 3x – y = 8.

21) Evaluate $\iint_R \cos\left(\dfrac{y - x}{y + x}\right) dA$ using a substitution if R is the trapezoid with vertices at (1, 0), (2, 0), (0, 1), and (0, 2).

22) Evaluate $\iint_R e^{x+y} \, dA$ using a substitution if R is the region $|x| + |y| \le 1$.

23) Find a map from $u^2 + v^2 \le 1$ onto $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} \le 1$ and then use it to find the area of the ellipse.

24) Compute the area of the region enclosed by the ellipse $x^2 + 2xy + 2y^2 - 4y = 8$ by completing squares and then making a substitution and integrating the appropriate integral.

25) CAS Problem (3 points): Use **MatLab** to solve the following. Use a live script and turn in a pdf copy.

Draw the portion of the sphere $x^2 + y^2 + z^2 = 9$ that is inside the paraboloid $x^2 + y^2 = 8z$ and approximate its surface area to four digits.

Brief answers:

1) $x - 2z = -1$ 2) $\dfrac{\pi\left(17^{3/2} - 5^{3/2}\right)}{6}$ 3) $18\sqrt{14}$ 4) $\dfrac{\pi}{3}$

5) $\dfrac{5\sqrt{5}}{48} + \dfrac{1}{240}$ 6) 16π 7) 12 grams 8) $\dfrac{\pi}{12}\left(17^{3/2} - 5^{3/2}\right)$ grams

9) $\dfrac{10\sqrt{10} - 1}{54}$ grams 10) 4π 11) $-4\pi^2$ 12) $\dfrac{-1}{12}$ 13) $\dfrac{-4\pi}{3}$

14) $\dfrac{-e}{2}$ 15) $2\pi + \dfrac{8}{3}$ 16) $\left(0, 0, \dfrac{a}{2}\right)$ 17) 4π 18) -3

19) $\ln(9)$ 20) $\dfrac{24\ln(2)}{5}$ 21) $\dfrac{3\sin(1)}{2}$ 22) $e - e^{-1}$

23) πab 24) 12π