Math 200
Homework #8: (21 problems)

Answers to every homework question are given so your homework grade will depend only on completeness and presentation. I should be able to see at a glance that you have done all of the problems with sufficient work. Write your problems in order and write the circled problem number to the left of the margin; draw the margin if necessary. If you run out of room in the middle of a problem, write a note directing me to the end of the assignment and put the remainder of your work there. Leave a margin at the top of each side of your stapled papers so that the problem numbers and all of your work are visible.

#1, 2: Calculate the curl of $\vec{F}$.

1) $\vec{F}(x,y,z)=\left\langle \frac{y}{x},\frac{y}{z},\frac{z}{x}\right\rangle$

2) $\vec{F}(x,y,z)=e^{y}\hat{\mathbf{i}}+\sin x\hat{\mathbf{j}}+\cos x\hat{\mathbf{k}}$

3) Verify Stokes' Theorem if $\vec{F}(x,y,z)=\left\langle y,x,x^2+y^2\right\rangle$ and S is the upper hemisphere $x^2+y^2+z^2=1$, $z\ge 0$ oriented up.

4) Use Stokes' Theorem to compute $\iint_S \nabla\times\vec{F}\cdot d\vec{S}$ if $\vec{F}(x,y,z)=\left\langle x+y,z^2-4,x\sqrt{y^2+1}\right\rangle$ and S is the wedge – shaped box without a top oriented out that is bounded by the coordinate planes, x + y = 1, and z = 2.

5) Use Stokes' Theorem to compute $\iint_S \nabla\times\vec{F}\cdot d\vec{S}$ if $\vec{F}(x,y,z)=x^2z^2\hat{\mathbf{i}}+y^2z^2\hat{\mathbf{j}}+xyz\hat{\mathbf{k}}$ and S is the part of the cone $y^2=z^2+x^2$ that lies between y = 0 and y = 3, oriented with a positive y – coordinate.

6) Use Stokes' Theorem to compute $\iint_S \nabla\times\vec{F}\cdot d\vec{S}$ if $\vec{F}(x,y,z)=\left\langle xyz,xy,x^2yz\right\rangle$ and S consists of the top and four sides, but not the bottom, of the cube with vertices $(\pm1,\pm1,\pm1)$, oriented outward.

#7 – 9: Use Stokes' Theorem to find $\int_C \vec{F}\cdot d\vec{s}$ if C is oriented counterclockwise from above.

7) $\vec{F}(x,y,z)=\left\langle x+y^2,y+z^2,z+x^2\right\rangle$ and C is the triangle with vertices (1, 0, 0), (0, 1, 0), and (0, 0, 1).

8) $\vec{F}(x,y,z)=yz\hat{\mathbf{i}}+2xz\hat{\mathbf{j}}+e^{xy}\hat{\mathbf{k}}$ and C is the intersection of z = 5 with $x^2+y^2=16$.

9) $\vec{F}(x,y,z)=\left\langle x^2z,xy^2,z^2\right\rangle$ and C is the intersection of 3x + 2y + z = 1 with $x^2+y^2=9$.

10) Calculate $\iint_S \nabla\times\vec{F}\cdot d\vec{S}$ if $\vec{F}$ satisfies the conditions of Stokes' Theorem and S is the sphere $\rho=a$.

#11, 12: compute the divergence of the given vector field.

11) $\vec{F}(x,y,z)=\left\langle x-2zx^2, z-xy, z^2x^2\right\rangle$

12) $\vec{G}(x,y,z)=\nabla\times\vec{F}(x,y,z)$ where $\vec{F}(x,y,z)=\left\langle F_1(x,y,z), F_2(x,y,z), F_3(x,y,z)\right\rangle$

13) Verify the Divergence Theorem for $\vec{F}(x,y,z)=2x\hat{\mathbf{i}}+3z\hat{\mathbf{j}}+3y\hat{\mathbf{k}}$ and the region $x^2+y^2\le 1$ for $0\le z\le 2$.

14) Use the Divergence Theorem to evaluate $\iint_S \left\langle x, y^2, z+y\right\rangle\cdot d\vec{S}$ if S is the boundary of the region contained in $x^2+y^2=4$ between z = x and z = 8. S is oriented out.

15) Use the Divergence Theorem to evaluate $\iint_S \left\langle \cos z+xy^2, xe^{-z}, \sin y+x^2z\right\rangle\cdot d\vec{S}$ if S is the boundary of the solid inside $z=x^2+y^2$ below z = 4, oriented out.

16) Use the Divergence Theorem to evaluate $\iint_S 3x+y+z^2\, dS$ if S is $x^2+y^2+z^2=9$.

17) Prove $\oiint_{\partial E}\left(f\nabla g\right)\cdot\hat{n}\,dS=\iiint_E f\nabla^2 g+\nabla f\cdot\nabla g\, dV$.

18) Prove $\oiint_{\partial E} D_{\hat{n}} f\, dS=\iiint_E \nabla^2 f\, dV$.

19) A hose feeds into a small screen box of volume 10 cm^3 that is suspended in a swimming pool. Water flows out across the surface of the box at a rate of 12 cm^3 / sec. Let $\vec{v}$ be the velocity field of water and P the center of the box. Estimate the divergence of $\vec{v}$ at P and include units in your final answer.

20) The velocity field of a fluid $\vec{v}$ cm / sec has a divergence at P = (2, 2, 2) of 3 / sec. Estimate the flow rate out of the sphere of radius 0.5 cm centered at P.

21) CAS Problem (3 points): Use **MatLab** to solve the following. Use a live script and turn in a pdf copy.

Verify the Divergence theorem if $\vec{F}(x,y,z)=\langle x^2y^2, y^2z^2, z^2x^2\rangle$ and the solid E is bounded by $x^2+y^2+z^2=2$ on top and $z=\sqrt{x^2+y^2}$ on the bottom.

Brief answers:

1) $\frac{y}{z^2}\hat{\mathbf{i}}+\frac{z}{x^2}\hat{\mathbf{j}}-\frac{1}{x}\hat{\mathbf{k}}$ 2) $\langle 0, \sin x, \cos x - e^y \rangle$ 3) 0

4) 1/2   5) 0   6) 0   7) -1 8) 80π

9) $243\pi/4$ 10) 0   11) $1-4zx-x+2zx^2$ 12) 0

13) 4π 14) 64π 15) $32\pi/3$ 16) 108π

19) $1.2\ \sec^{-1}$ 20) $\pi/2\ \ cm^3/\sec$