Math 200
Homework #9: (19 problems)

Answers to every homework question are given so your homework grade will depend only on completeness and presentation. I should be able to see at a glance that you have done all of the problems with sufficient work. Write your problems in order and write the circled problem number to the left of the margin; draw the margin if necessary. If you run out of room in the middle of a problem, write a note directing me to the end of the assignment and put the remainder of your work there. Leave a margin at the top of each side of your stapled papers so that the problem numbers and all of your work are visible.

#1 – 7: find and classify the critical points for each function.

1) $f(x,y) = x^2 + 2y^2 - 4y + 6x$

2) $g(x,y) = x^2 - 12xy + y$

3) $h(x,y) = x^3 + y^4 - 6x + 2y^2$

4) $f(x,y) = xye^{-x^2-y^2}$

5) $h\left(x,y\right) = \ln x + 2\ln y - x - 4y$

6) $f(x,y) = x - y^2 - \ln(x+y)$

7) Find the maximum volume of a box inscribed in the tetrahedron bounded by the coordinate planes and by $x + \frac{y}{2} + \frac{z}{4} = 1$. “Inscribed” means the box has a side in each of the three coordinate planes, and one vertex laying on the plane $x + \frac{y}{2} + \frac{z}{4} = 1$.

8) Find the point on the plane z = x + y + 1 closest to P = (1, 0, 0).

#9 – 11: classify all critical points of the given function with given constraint. Draw level curves and explain how your answer is consistent with your contour plot.

9) $f(x,y) = 2x + 3y$ constrained by $x^2 + y^2 = 4$.

10) $p(x,y) = y^2 - x$ constrained by $x^2 + 4y^2 = 36$.

11) $g(x,y) = 4x^2 + 9y^2$ constrained by $xy = 4$.

#12 – 14: find the constrained global extreme values of the given constrained function.

12) $h(x,y)=x^2y^4$ constrained by $x^2+2y^2=6$.

13) $f(x,y,z)=3x+2y+4z$ constrained by $x^2+2y^2+6z^2=1$.

14) $g(x,y,z)=xy+3xz+2yz$ constrained by 5x + 9y + z = 10.

15) Find the point (a, b) on the graph of $y=e^x$ so that ab is the smallest number possible. Draw level curves and explain how your answer is consistent with your contour plot.

16) Find the point on the ellipse $x^2+6y^2+3xy=40$ with the largest x – coordinate.

17) Find the maximum value of $h(x,y)=x^3y^2$ on the unit circle.

18) Find the maximum value of $g(x,y,z)=x+y+z$ subject to the constraints $x^2+y^2+z^2=9$ and $\frac{x^2}{4}+\frac{y^2}{4}+4z^2=9$.

19) CAS Problem (3 points): Use **MatLab** to solve the following. Use a live script and turn in a pdf copy.

Use the second partials test to classify all critical points of $f(x,y)=x^4-3xy+2y^2$ as minimums, maximums, or saddles. You do not need to determine local or absolute extrema.

Brief answers:

1) $(-3,1)$ is a global minimum.

2) $\left(\frac{1}{12},\frac{1}{72}\right)$ is a saddle.

3) $\left(\sqrt{2},0\right)$ is a local minimum and $\left(-\sqrt{2},0\right)$ is a saddle.

4) (0, 0) is a saddle; $\left(\frac{1}{\sqrt{2}},\frac{1}{\sqrt{2}}\right)$ and $\left(\frac{-1}{\sqrt{2}},\frac{-1}{\sqrt{2}}\right)$ are global maximums; $\left(\frac{-1}{\sqrt{2}},\frac{1}{\sqrt{2}}\right)$ and $\left(\frac{1}{\sqrt{2}},\frac{-1}{\sqrt{2}}\right)$ are global minimums.

5) $\left(1,\frac{1}{2}\right)$ is a global maximum.

6) $\left(\frac{3}{2},\frac{-1}{2}\right)$ is a saddle.

7) Maximum volume is $\frac{8}{27}$ cubic units.

8) The point $\left(\frac{1}{3},\frac{-2}{3},\frac{2}{3}\right)$ is closest.

9) $\left(\frac{4}{\sqrt{13}},\frac{6}{\sqrt{13}}\right)$ is a global maximum, and $\left(\frac{-4}{\sqrt{13}},\frac{-6}{\sqrt{13}}\right)$ is a global minimum.

10) $\left(-2, \pm 2\sqrt{2}\right)$ are global maximums, $(6,0)$ is a global minimum, and $(-6,0)$ is a local minimum.

11) Both $\left(\sqrt{6}, \frac{2\sqrt{6}}{3}\right)$ and $\left(-\sqrt{6}, \frac{-2\sqrt{6}}{3}\right)$ are global minimums.

12) 0 is the global minimum constrained value and 8 is the global maximum constrained value.

13) $\sqrt{\frac{41}{3}}$ is the global maximum constrained value and $-\sqrt{\frac{41}{3}}$ is the global minimum constrained value.

14) $g(-5/3, 20/9, -5/3) = \frac{-25}{9}$ is not a global minimum (since g(4,0,-10) = -120) nor a global maximum (because $g(0,0,10)$ =0) so there is no global constrained minimum or maximum.

15) $\left(-1, e^{-1}\right)$ is the point with smallest constrained value of xy.

16) The largest x – coordinate is 8.

17) The largest value is $\frac{2 \cdot 3^{3/2}}{5^{5/2}}$.

18) The maximum value of the constrained function is $\frac{3+6\sqrt{2}}{\sqrt{5}}$.