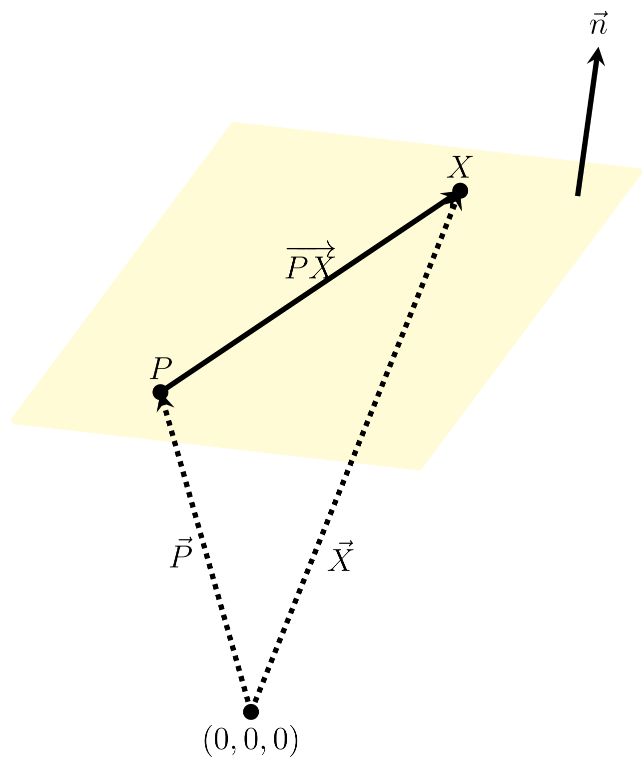
Surfaces (HW #1)

Planes

Given a point P in a plane and a vector $\vec{n} = \langle a, b, c \rangle$ perpendicular to the plane we will derive the standard form for the equation of a plane:

$$ax + by + cz = d.$$

To do this, let $X = (x, y, z)$ be an arbitrary point in the plane and draw the vector $\overrightarrow{PX}$ in the plane. $\overrightarrow{PX}$ is perpendicular to $\vec{n}$.



But then

$$\overrightarrow{PX} \cdot \vec{n} = 0$$

$$\implies (\vec{X} - \vec{P}) \cdot \vec{n} = 0$$

$$\implies \vec{X} \cdot \vec{n} = \vec{P} \cdot \vec{n}$$

$$\implies \langle x, y, z \rangle \cdot \langle a, b, c \rangle = d$$

where $d = \vec{P} \cdot \vec{n}$. Multiplying gives the standard form of the plane

$$ax + by + cz = d.$$

Find an equation for the plane in standard form that contains the three points $P = (2, 5, 7)$, $Q = (1, -1, -2)$, and $R = (1, 1, 3)$.

Sketch a graph for $x + 2y + 3z = 6$ using intercepts.

Coordinate planes are of the form $x = d$, $y = d$, or $z = d$. Sketch $x = 3$, $y = 2$, or $z = 4$ on three separate sets of axes. Dot the axes first and then fill in the lines later.

Spheres

Find the equation of the sphere with center at the point $(2, -5, 3)$ and with radius $R = 8$.

The sphere centered at (a, b, c) with radius R is $(x - a)^2 + (y - b)^2 + (z - c)^2 = R^2$. The basic untranslated sphere with center at the origin is then

$$x^2 + y^2 + z^2 = R^2$$

. The unit sphere has $R = 1$.

The sphere is an example of a quadratic surface since the left side is a polynomial of degree two. There is a group of basic quadratic surfaces for which I ask you to please know:
1) The equation that corresponds to each surface.
2) How to sketch the surfaces.
3) How to label the axes with positive orientation ($\hat{\imath}, \hat{\jmath}, \hat{k}$ oriented by the right-hand rule.)
4) Do # 1 - 3 above if the variables are permuted in the equations.

Sketch $x^2 + y^2 + z^2 = 9$. Notice the equation does not change if the variables are permuted; this indicates symmetry about any line or plane passing through $(0, 0, 0)$. **Traces** of a surface are the curves we get when setting one of the variables to a constant. Sketch some traces on the sphere.

Sketch the following quadratics with axes that are positively oriented.

1)**Cylinders**: Sketch $x^2 + y^2 = R^2$ where $R > 0$.

How does the sketch change for $z^2 + y^2 = R^2$? A variable is **free** if it is not in an equation, as in "free from any constraint."

The idea of a cylinder can be generalized to any equation with a variable missing. This is called a **generic cylinder**. For instance, sketch $y = x^2$.

2) **Paraboloids**: Sketch $z = x^2 + y^2$. How is the graph of $z = -x^2 - y^2$ different?

3) **Saddles**: Sketch $z = x^2 - y^2$.

4) **Cones**: Sketch $z^2 = x^2 + y^2$. This is a double cone. Find equations for the top and bottom cones.

We can pull the two cones apart with $z^2 - 1 = x^2 + y^2$ and push them into a nuclear reactor shape with $z^2 + 1 = x^2 + y^2$. Notice z is constrained in the first equation, $|z| \geq 1$, but not in the second.