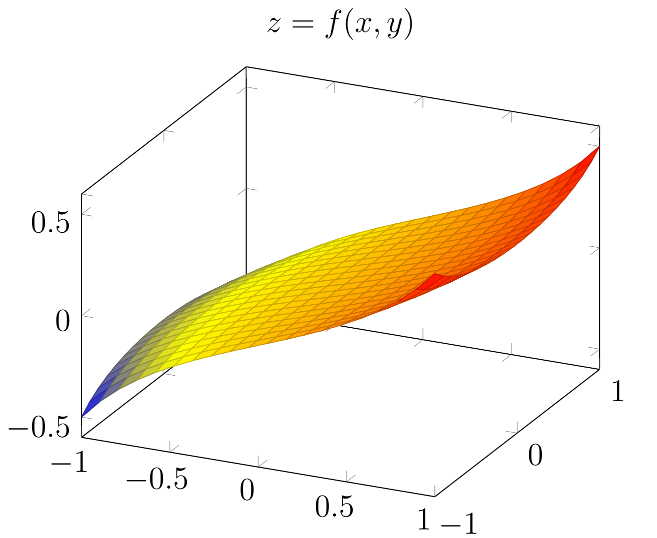
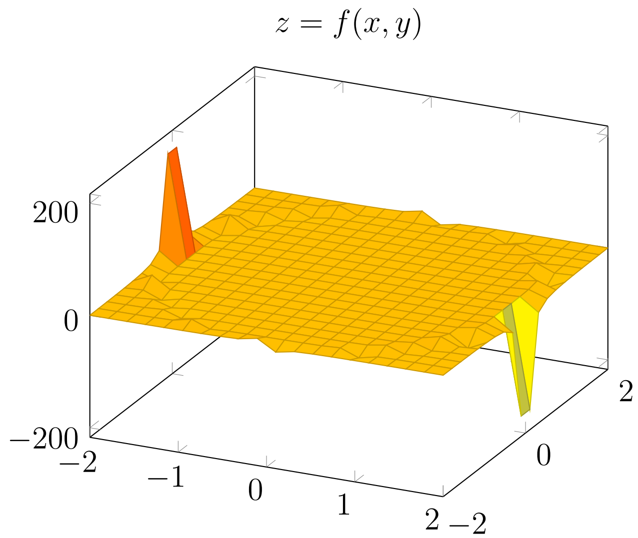
# Functions With More Than One Input (HW #2)

$f(x, y) = \dfrac{x}{4 - x^2 - y^2}$ $\qquad$ $f(1, 0) =$ ________________ $\qquad$ Domain = ________________

Draw a shaded region equal to the domain.

The graph of $f(x, y)$ is the surface $z = f(x, y)$. $z$ is the **dependent variable** or **output**; $x$ and $y$ are the **independent variables** or **inputs**. I wouldn't ask you to graph this by hand. Here it is, one graph with domain restricted to $x$ and $y$ between $-1$ and $1$, and the other unrestricted.



Sketch the graph of $h(x, y) = -x^2 + 2x - y^2$. Drawing the dotted axes first may help.

We can have more than two inputs.

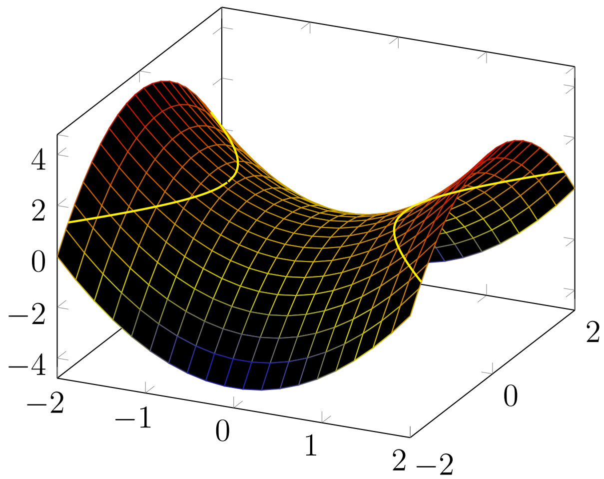
$f(x,y,z) = \dfrac{y+z}{16-x^2-y^2-z^2}$ $f(1,2,3) =$ ________________ Domain $=$ ________________

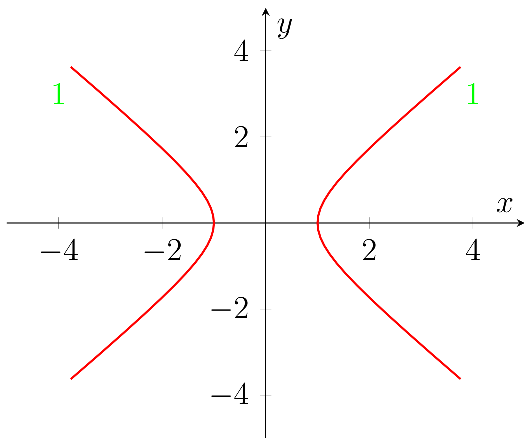
Draw a shaded solid region equal to the domain.

The graph of $f(x,y,z)$ would be the graph of the **hypersurface** $w = f(x,y,z)$, but we would need four dimensions to draw it. Instead we develop the idea of a **contour plot** where sections of the graph for which the independent variable is held constant are projected into the domain. This idea works best for two inputs - topo maps are made this way - so we start with such a function and work our way towards three inputs after.

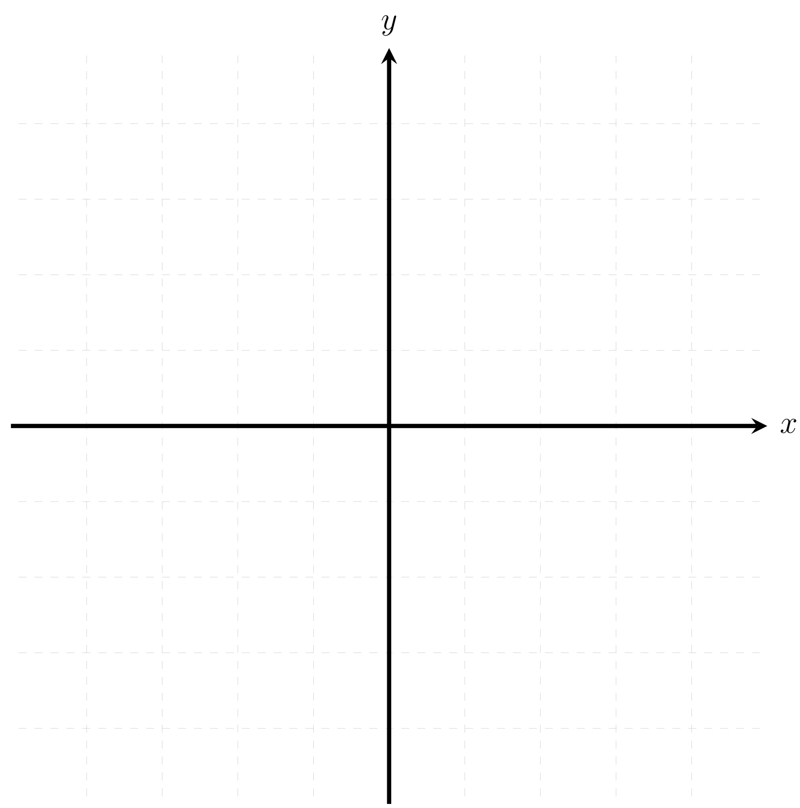
Draw a contour plot for $g(x, y) = x^2 - y^2$.

The intersection of $z = 1$ and $z = x^2 - y^2$ is the two yellow curves on the saddle. Project these curves onto the $xy$-plane and mark them with the number 1 for $z = 1$. That is, graph $1 = x^2 - y^2$ and label it appropriately. $1 = x^2 - y^2$ is a **level curve** for the function. Repeat this process for different values of $z$ producing several level curves.




Finish the contour plot for $g(x, y)$ below. Write the level curves used on the left.



Sketch a contour plot for $g(x, y, z) = \dfrac{1}{x^2 + y^2 + z^2}$.