Limits (HW #3)

$\lim_{(x,y)\to(a,b)} f(x,y) = L$ means $f(x,y)$ gets as close as we wish to L as (x,y) gets close to (a,b).

For those of you who like rigor you might find something like this in a book:

$$\lim_{(x,y)\to(a,b)} f(x,y) = L \iff \forall \epsilon > 0 \ \exists \ \delta > 0 \ \ni \ |f(x,y) - L| < \epsilon \text{ whenever } 0 < \|\langle x,y\rangle - \langle a,b\rangle\| < \delta.$$

Our goal is to calculate limits. To do that, we have to agree on what are the known continuous functions because limits of continuous functions are easy to calculate by definition.

$f(x,y)$ is **continuous** at (a,b) if and only if $\lim_{(x,y)\to(a,b)} f(x,y) = f(a,b)$.

We assume all familiar functions that have unbroken graphs from Calculus are continuous, and that any sum, product, or composition of continuous functions is also continuous.

Find $\displaystyle\lim_{(x,y)\to(1,0)} \frac{x\cos(\pi y) - \ln(1 + x^2y^2)}{2^x(y^2+1)}$.

Path Theorem: Suppose $\lim_{(x,y)\to(a,b)} f(x,y) = L$, $\vec{s}(t) = \langle x(t), y(t)\rangle$, and $\vec{s}(t) \to (a,b)$ as $t \to k$. Then

$$\lim_{t\to k} f(x(t), y(t)) = L$$

.

Draw a diagram illustrating this theorem. Warning! This theorem is never used to calculate limits, but is used to prove they don't exist.

Does $L = \lim_{(x,y)\to(0,0)} \dfrac{xy + x^2}{x^2 + y^2}$ exist? Defend your answer. Notice substituting in $(0,0)$ gives and indeterminate form. There is no analoguous theorem for L'Hospital's rule in multivariable calculus.

Product Limit Theorem: If $\lim_{(x,y)\to(a,b)} f(x,y) = L$ and $\lim_{(x,y)\to(a,b)} g(x,y) = K$, then

$$\lim_{(x,y)\to(a,b)} f(x,y)g(x,y) = LK$$

Find $L = \lim_{(x,y)\to(0,0)} \dfrac{(y+1)\sin(x)}{x(y+2)}$ if it exists; otherwise, prove it does not exist. Notice substituting in $(0,0)$ gives and indeterminate form.

Find the following limits if they exist or show why they don't exist.

$L = \lim_{(x,y)\to(0,0)} \dfrac{x^3}{x^2+y^2}$ Hint: convert to polar coordinates.

$$L = \lim_{(x,y)\to(0,0)} \frac{6x^3y}{2x^4+y^4}$$

$$L = \lim_{(x,y)\to(\infty,\infty)} \frac{\ln(y)x}{ye^x}$$