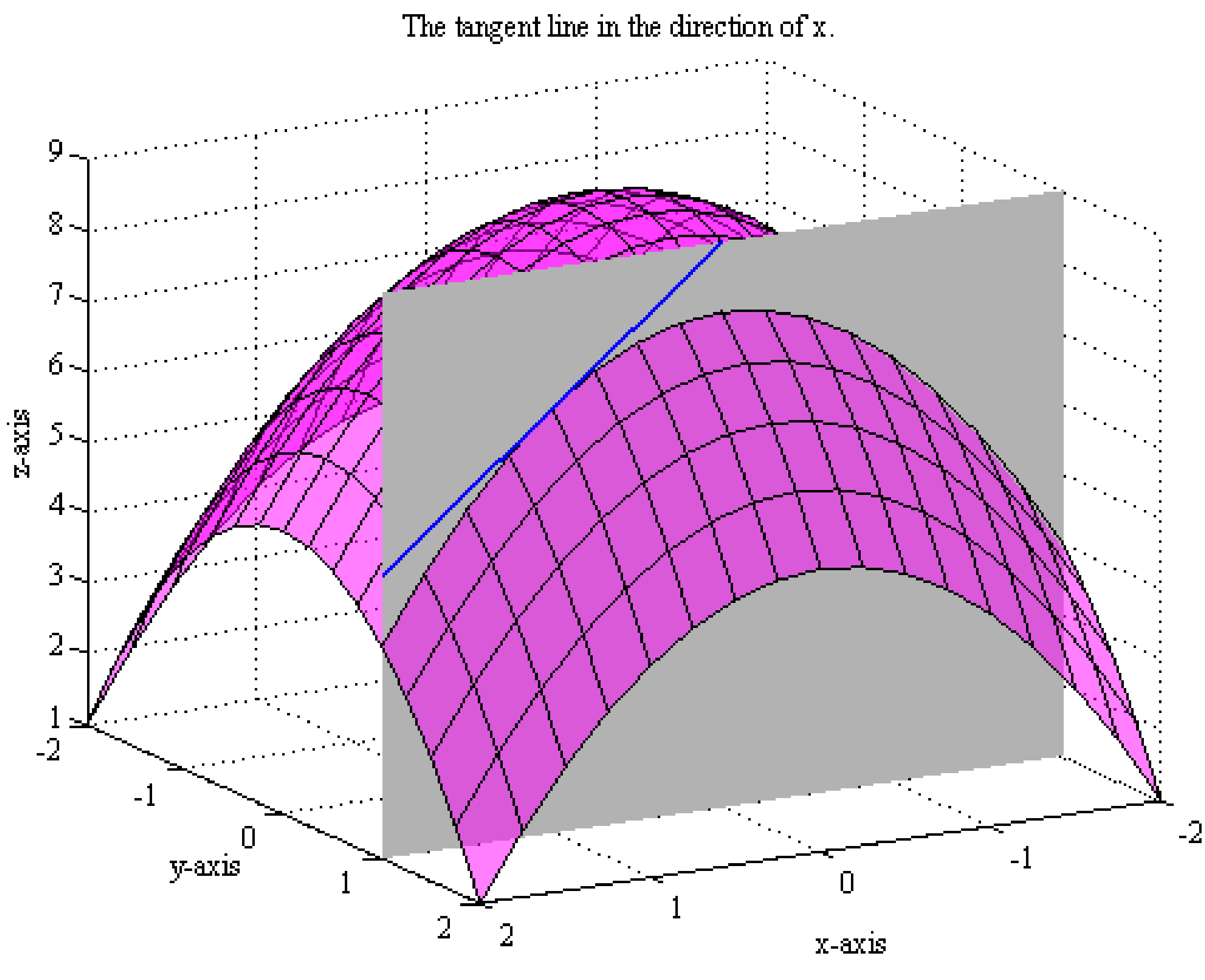
Partial Derivatives (HW #3)

$$\frac{\partial f(x,y)}{\partial x} = \lim_{h \to 0} \frac{f(x+h,y) - f(x,y)}{h}$$

is the **partial derivative of $f(x,y)$ with respect to x**. All the other input variables are held constant as x varies.

Alternate notation: $f_x(x,y)$. The partial derivative is the slope of a particular tangent line to the graph of $f(x,y)$. Below is the picture of a tangent line with slope equal to $f_x(1,1)$. Alternate notation: $\left.\frac{\partial f}{\partial x}\right|_{(1,1)}$.



Write the definition and notation for the **partial derivative of $f(x,y)$ with respect to y**.

Calculating partial derivatives is easy. Hold all but one input variable constant and differentiate as you did in single variable calculus with respect to that one variable that varies.

Find the first order partial derivatives of $f(x, y) = xy^2 - xy - x^2 + y^2$.
What is $f_y(1, 1)$? What is $\left.\frac{\partial f}{\partial x}\right|_{(1,1)}$?

The **gradient** of $f(x, y)$ is

$$\nabla f = \left\langle \frac{\partial f}{\partial x}, \frac{\partial f}{\partial y} \right\rangle .$$

The gradient of f is a vector with components equal to the first order partial derivatives of f. $\nabla = \left\langle \frac{\partial(*)}{\partial x}, \frac{\partial(*)}{\partial y} \right\rangle$ is an **operator** just as $\frac{d(*)}{dx}$ and $\int_0^t (*)\, dx$ are - they all have functions as inputs placed where the $(*)$ is.

What is the gradient of $f(x, y) = xy^2 - xy - x^2 + y^2$?
What is $\nabla f(1, 1)$?

Find the gradient for $T(x, y, z) = 100 - \rho$, where ρ is the first spherical coordinate.

We can find higher order partial derivatives. When doing this we need only remember the mixed partial's theorem: **mixed partials are equal if the mixed partial derivatives of f are continuous in a rectangle about the point in question.** Since most points of most functions you know of meet this condition, you rarely need to worry about what order you differentiate.

Notation: $f_{xy} = f_{yx}$ or $\dfrac{\partial^2 f}{\partial x \partial y} = \dfrac{\partial^2 f}{\partial y \partial x}$, where $f_{xy} = \dfrac{\partial^2 f}{\partial y \partial x} = \dfrac{\partial \left(\frac{\partial f}{\partial x}\right)}{\partial y}$.

$g(x, y, z) = \ln(x^2 yz) - xy^2 z^2$. Find all of its second partial derivatives. Which input variable should we increase by 0.1 at the point $(-1, 3, 1)$ to get the largest increase in the output of g? Hint: Compare the rates of change at $(-1, 3, 1)$.

Estimate the first order partial derivatives at $(0.6, 0.6)$ for the function $h(x, y)$ that has the following contour plot.