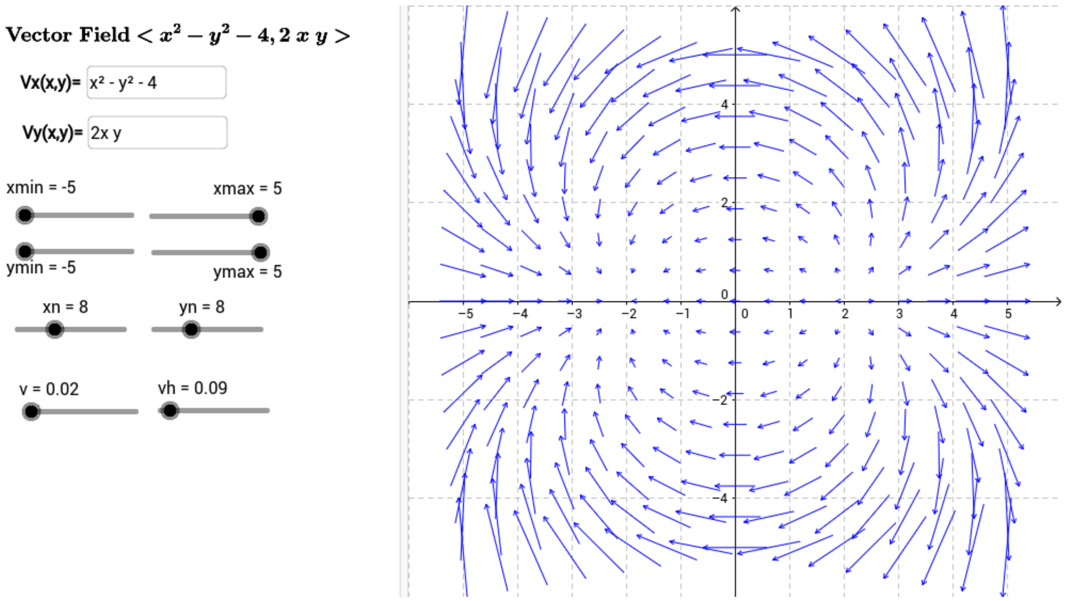
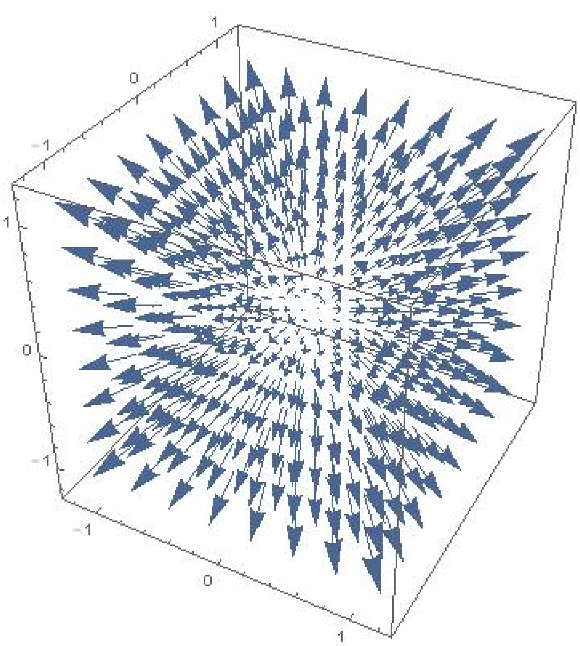
Vector Fields (HW #3)

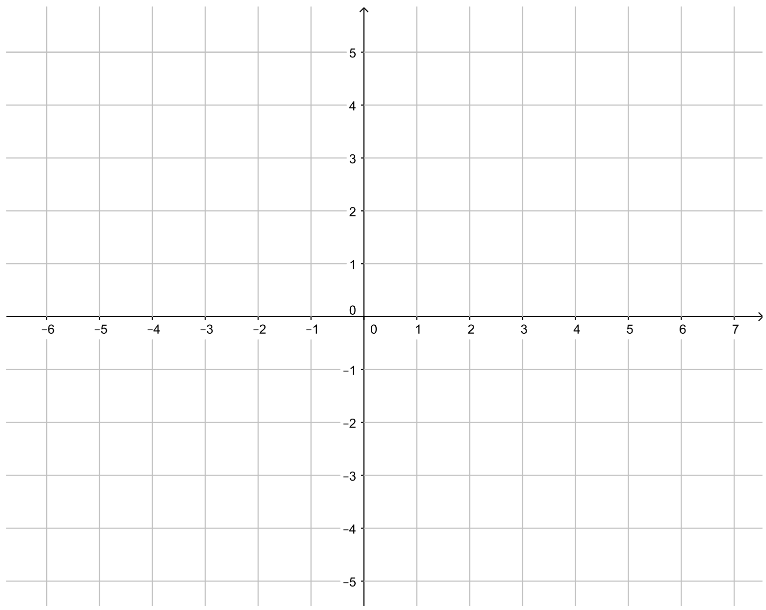
A **Vector Field** is a function that assigns a vector to each point in the input space; but we have seen that already with vector functions. In this case, the book assumes the domain is $\mathbb{R}^n$. So for $n = 3$, $\vec{F}(x, y, z) = \langle F_1(x, y, z), F_2(x, y, z), F_3(x, y, z)\rangle$ is a vector function. A graphical representation draws arrows at a selection (or mesh) of points in the input space.



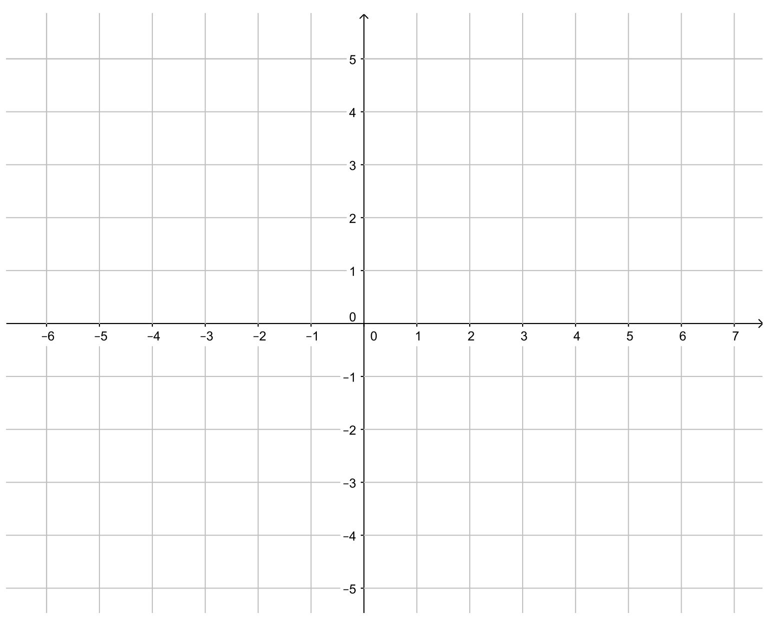


Look at https://earth.nullschool.net/ for a vector field with points on earth's surface as the set of inputs and vector outputs representing wind currents. Zoom in towards your favorite location on earth.

A constant vector field draws the same vector at each point. Sketch $\vec{F}(x, y) = \langle 1, 2\rangle$. Find the work done by this constant vector field on a particle that moves from $(0, -4)$ to $(4, 0)$. Sketch the displacement on the graph of your vector field.



Is the work done by $\vec{G}(x, y) = (1 - y)\hat{\imath} + x\hat{\jmath}$ on a particle that moves from $(2, 0)$ to $(-2, 0)$ on the semicircle $y = \sqrt{4 - x^2}$ positive, negative, or zero? Make a sketch to defend your answer.



Can we differentiate vector fields? Yes! There are two different derivatives of vector fields that I want you to be able to calculate: the **curl**$\vec{F} = \nabla \times \vec{F}$ and the **divergence** of $\vec{F} = \text{div } \vec{F} = \nabla \cdot \vec{F}$. We will talk more about the meaning of these after we talk about surface integrals; for now I want you to be able to calculate them.

$\nabla \times \vec{F}$ is a vector calculated much like a cross product. Calculate the curl of $\vec{F} = \langle 3x^2 - y, 2y + xz, \sin(z) - x^2 \rangle$.

$\nabla \cdot \vec{F}$ is a scalar calculated much like a dot product. Calculate the divergence of $\vec{F} = \langle 3x^2 - y, 2y + xz, \sin(z) - x^2 \rangle$.

Review your calculations and notice which terms in each component have nonzero contributions to the end result.