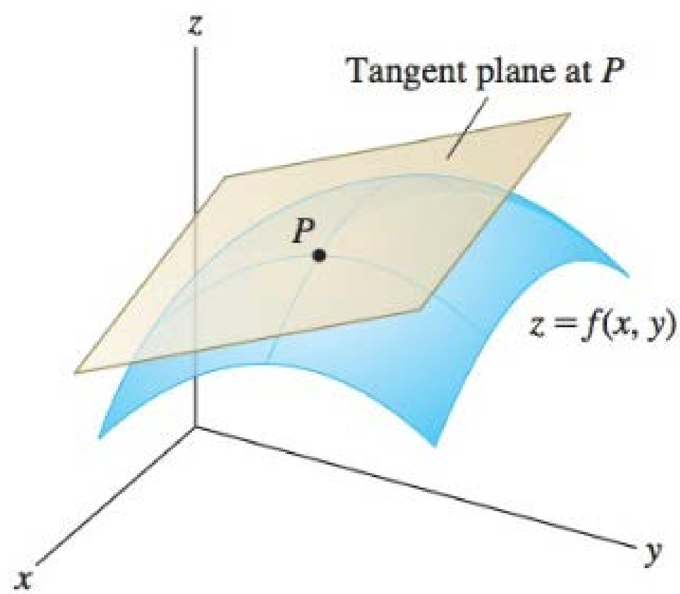
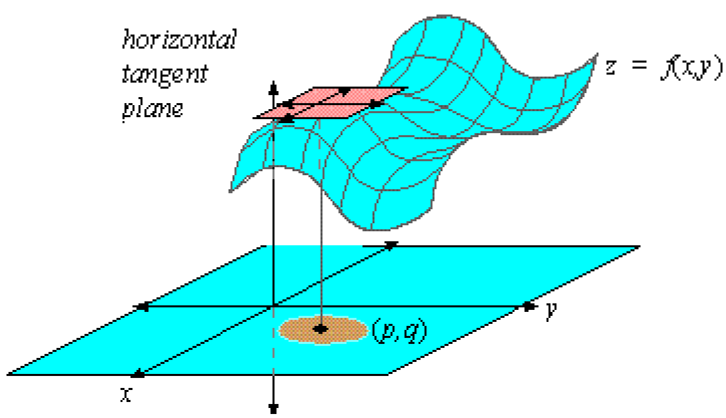
Tangent Plane and Differential (HW #4)

If a function $f(x,y)$ has continuous partial derivatives in a disk about (a,b), then there exists a tangent plane that gives the best linear approximation to $f(x,y)$ at the point P $=(a,b,f(a,b))$.



Here is another picture of a horizontal tangent plane located at $(p,q,f(p,q))$.



If z = f(x,y) how can we find the equation for the tangent plane to the graph at the point P = $(a,b,f(a,b))$?

The two first partial derivatives allow us to construct two vectors parallel to the tangent plane at P. $\langle 1,0,f_x \rangle$ and $\langle 0,1,f_y \rangle$. Draw these vectors on the first picture above with tail at P.

Now derive the equation for the tangent plane.

$$z - f(a,b) = f_x(a,b)(x-a) + f_y(a,b)(y-b).$$

is analogous to the point-slope form of a line. We can also write

$$z - f(a,b) = f_x \Delta x + f_y \Delta y \qquad \text{or} \qquad z = f(a,b) + f_x \Delta x + f_y \Delta y$$

where $\Delta x = x - a$ and $\Delta y = y - b$ are the changes in x and y respectively.

Find the equation for the tangent plane to the surface z = $g(x, y) = 2x + y + \ln(xy)$ at the point (1, 1, 3). Write your answer in $ax + by + cz = d$ form.

Define the change in the output **on the tangent plane** to be the **differential**

$$df = f_x \Delta x + f_y \Delta y \, .$$

The differential is rigorously defined in more advanced integration theory and written

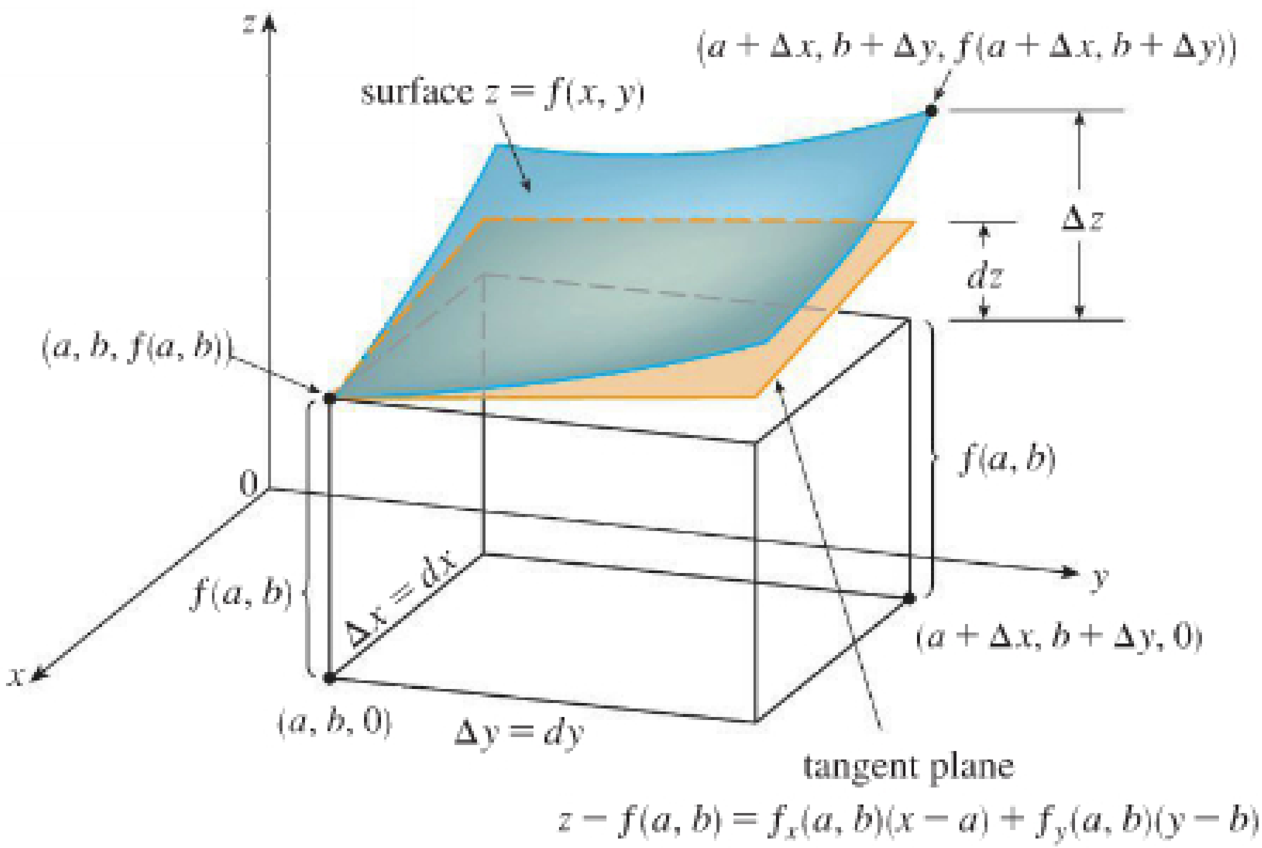
$$df = f_x dx + f_y dy$$

and so both are used interchangeably in this class with the understanding that $\Delta x = dx$ and $\Delta y = dy$.

Compare this with the change in output **on the graph of f,**

$$\Delta f = f(x,y) - f(a,b)$$

These are not the same!!! Label df and Δf in the following picture.



You are meant to remember that as Δx and Δy get small,

$$df \approx \Delta f \ .$$

This is brief notation to mean that the tangent plane is a nice approximation for the surface if we stay close to (a, b). In fact, if the graph of $L(x, y)$ equals the tangent plane at the point $(a, b, f(a, b))$, then $L(x, y)$ is called the **linear approximation** of $f(x, y)$. So

$$L(x, y) = f(a, b) + f_x(a, b)\Delta x + f_y(a, b)\Delta y \ .$$

Use differentials to estimate $f(2.9, 0.2)$ if $f(x, y) = \sqrt{x + e^{4y}}$. What is the linear approximation for $f(x, y)$ at (a, b)?

$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$ relates the total resistance R of two resistances placed parallel to each other; estimate the maximum error for R using this formula if the measurements of $R_1 = 20\,\Omega$ and $R_2 = 50\,\Omega$ with a maximum 1% error. Hint: differentiate implicitly.