Quiz 1 25 points Math 200 Name: ______________________

1. (6 points) Find the equation of the plane in standard form that contains the points $P = (1, 1, -2)$, $Q = (2, 3, 1)$, and $R = (4, -1, 1)$.

$\vec{PQ} = \langle 1, 2, 3 \rangle$

$\times\ \vec{PR} = \langle 3, -2, 3 \rangle$

$\langle 12, 6, -8 \rangle$

Use $\vec{n} = \langle 6, 3, -4 \rangle$

and $\vec{P} = \langle 1, 1, -2 \rangle$.

Let $\vec{X} = (x, y, z)$ be any point in the plane.

Then $\vec{X} \cdot \vec{n} = \vec{P} \cdot \vec{n}$

$\Rightarrow 6x + 3y - 4z = \langle 1, 1, -2 \rangle \cdot \langle 6, 3, -4 \rangle$

$\Rightarrow \boxed{6x + 3y - 4z = 17}$

2. Let $\vec{u} = 2\hat{\imath} - 3\hat{\jmath} - \hat{k}$ and $\vec{p} = \langle 3, 0, 4 \rangle$. You may use work from one part in other parts.

(a) (4 points) Find the area of the rectangle spanned by $\vec{p}$ and $\vec{u}$.

$\langle 2, -3, -1 \rangle$

$\times\ \langle 3, 0, 4 \rangle$

$\langle -12, -11, 9 \rangle$

$\therefore$ area $= \| \langle -12, -11, 9 \rangle \|$

$= \sqrt{144 + 121 + 81}$

$= \boxed{\sqrt{346}}$

(b) (2 points) Find the flux of the constant vector field $\vec{F} = \langle -2, 1, 1 \rangle$ through the parallelogram spanned by $\vec{u}$ and $\vec{p}$ and oriented from $\vec{u}$ to $\vec{p}$.

Flux $= \vec{F} \cdot (\vec{u} \times \vec{p}) = \langle -2, 1, 1 \rangle \cdot \langle -12, -11, 9 \rangle = 24 - 11 + 9 = \boxed{22}$

(c) (1 point) Find the volume of the box spanned by $\vec{F}$, $\vec{p}$, and $\vec{u}$.

Vol $= |\text{Flux}| = \boxed{22}$

Q1

3. Let $\vec{v} = \hat{\imath} - 2\hat{\jmath} + 2\hat{k}$ and $\vec{w} = < 3, -1, 2 >$. You may use work from one part in other parts.

(a) (3 points) Find the cosine of the angle between $\vec{v}$ and $\vec{w}$.

$\vec{v} \cdot \vec{w} = \|\vec{v}\| \, \|\vec{w}\| \cos\theta$

$\Rightarrow \cos\theta = \dfrac{3+2+4}{\sqrt{9}\sqrt{14}}$

So $\boxed{\cos\theta = \dfrac{3}{\sqrt{14}}}$

(b) (4 points) Find $\vec{w}_{\|\vec{v}}$ and $\vec{w}_{\perp\vec{v}}$.

$\vec{w}_{\|\vec{v}} = \dfrac{\vec{w}\cdot\vec{v}}{\vec{v}\cdot\vec{v}}\vec{v}$

$\Rightarrow \vec{w}_{\|\vec{v}} = \dfrac{9}{9}\langle 1,-2,2\rangle$

$= \boxed{\langle 1,-2,2\rangle} = \vec{v}.$

$\vec{w}_{\perp\vec{v}} = \vec{w} - w_{\|\vec{v}}$

$= \langle 3,-1,2\rangle - \langle 1,-2,2\rangle$

$\Rightarrow \vec{w}_{\perp\vec{v}} = \boxed{\langle 2,1,0\rangle}$

(c) (1 point) Find the work done by a force $\vec{w}$ Newtons applied to a particle with displacement $\vec{v}$. meters

$W = \vec{w}\cdot\vec{v} = \boxed{9 \text{ joule}}$

(d) (4 points) Find a position function ~~or~~ And the coordinate equations of a line that passes through the point $(0, 3, -1)$ that is parallel to $\vec{w}$.

$\vec{\alpha}(t) = \langle 0,3,-1\rangle + t\langle 3,-1,2\rangle$

$\Rightarrow \boxed{\vec{\alpha}(t) = \langle 3t, 3-t, -1+2t\rangle}$

and

$$\boxed{\begin{aligned} x &= 3t \\ y &= 3-t \\ z &= -1+2t \end{aligned}}$$