Quiz 2 25 points Math 200 Name:

1. Let $\vec{\alpha}(t) = \langle t, \cosh(t), \sinh(t)\rangle$ for all parts of this problem. You do not need to repeat work.

(a) (5 points) Find the arc length of the trace of $\vec{\alpha}(t)$ from $t = 0$ to $t = \ln(2)$. Recall $\cosh^2(t) - \sinh^2 t = 1$

$$\vec{\alpha}'(t) = \langle 1, \sinh(t), \cosh(t)\rangle \Rightarrow \|\vec{\alpha}'(t)\| = \sqrt{1+\sinh^2 t+\cosh^2 t} = \sqrt{2}\cosh(t)$$

$$\Rightarrow S = \int_0^{\ln(2)} \sqrt{2}\cosh(t)\,dt = \sqrt{2}\sinh(t)\Big|_0^{\ln(2)}$$

$$= \boxed{\sqrt{2}\sinh(\ln(2))}$$

OR $\sqrt{2}\cdot\dfrac{2-\frac{1}{2}}{2} = \boxed{\dfrac{3\sqrt{2}}{4}}$

(b) (4 points) Find the equation of the osculating plane for the trace of $\vec{\alpha}(t)$ at $t = 0$.

$$\vec{\alpha}'(0) = \langle 1, 0, 1\rangle$$

$$\vec{\alpha}''(0) = \langle 0, \cosh(t), \sinh(t)\rangle\Big|_{t=0} = \langle 0, 1, 0\rangle.$$

$$\begin{array}{r} \langle 1,0,1\rangle \\ \times\ \langle 0,1,0\rangle \\ \hline \langle -1,0,1\rangle \end{array}$$

point $= \alpha(0) = (0,1,0)$.

use $\vec{n} = \langle 1,0,-1\rangle$

$\therefore$ the plane is

$$\boxed{x - z = 0}$$

(c) (4 points) Find the curvature for the trace of $\vec{\alpha}(t)$ at $t = 0$.

$$\kappa = \frac{\|\vec{v}\times\vec{a}\|}{v^3} = \frac{\|\langle -1,0,1\rangle\|}{(\sqrt{2})^3} = \frac{\sqrt{2}}{2\sqrt{2}} = \boxed{\frac{1}{2}}$$

Q2

2. (4 points) Find and simplify $I = \int_{-\pi}^{\pi} \langle t\sin(2t), \cos^2(\cancel{\pi}t), \sin(2\pi t)\cos(3\pi t)\rangle\, dt$.

even ODD

$$I = \left\langle 2\int_0^{\pi} t\sin(2t)\,dt,\ \int_{-\pi}^{\pi} \frac{1+\cancel{\cos(2t)}}{2}\,dt,\ 0\right\rangle$$

$\to 0;\ 2\pi$

$$\begin{array}{c|c|c} \oplus & t & \sin(2t) \\ \ominus & 1 & \frac{-\cos(2t)}{2} \\ \hline & 0 & \frac{-\sin(2t)}{4} \end{array}$$

$$\Rightarrow I = \left\langle 2\cdot\left[\frac{-t\cos(2t)}{2} + \cancel{\frac{\sin(2t)}{4}}^{\,0}\right]_0^{\pi},\ \pi,\ 0\right\rangle$$

$$\Rightarrow \boxed{I = \langle -\pi, \pi, 0\rangle}$$

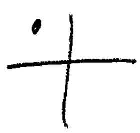
3. (4 points) Find $f'(0)$ if $f(t) = \vec{p}(t)\cdot\langle e^t, \sin(t), \arctan(t)\rangle$, $\vec{p}(0) = \langle -2, 1, 3\rangle$, and $\left.\frac{d\vec{p}}{dt}\right|_{t=0} = \langle 2, 0, 4\rangle$.

$$f'(0) = \langle 2,0,4\rangle\cdot\langle 1,0,0\rangle + \langle -2,1,3\rangle\cdot\left.\left\langle e^t, \cos(t), \frac{1}{1+t^2}\right\rangle\right|_{t=0}$$

$$= 2 + \langle -2,1,3\rangle\cdot\langle 1,1,1\rangle = 2+2$$

$$= \boxed{4}$$

4. (a) (2 points) Convert $P = (-1, 1, -3)$ to cylindrical coordinates.



2nd Quad

$$\left.\begin{array}{l} r^2 = 2 \Rightarrow r = \sqrt{2} \\ \tan\theta = \frac{1}{-1} = -1 \Rightarrow \theta = -\frac{\pi}{4} + \pi \end{array}\right\} \Rightarrow \boxed{P = \left(\sqrt{2}, \tfrac{3\pi}{4}, -3\right)_c}$$

2nd quad

(b) (2 points) Convert $P = (-1, 1, -3)$ to spherical coordinates.

$$\left.\begin{array}{l} \rho^2 = 2+9 = 11 \Rightarrow \rho = \sqrt{11} \\ \cos\phi = \frac{-3}{\sqrt{11}} \Rightarrow \phi = \cos^{-1}\left(\frac{-3}{\sqrt{11}}\right) \end{array}\right\} \Rightarrow \boxed{P = \left(\sqrt{11}, \tfrac{3\pi}{4}, \cos^{-1}\left(\tfrac{-3}{\sqrt{11}}\right)\right)_S}$$

(between $\frac{\pi}{2}$ and π)