

1. Let $f(x, y) = (x - y) \ln(x + y)$. You do not need to repeat work.

(a) (4 points) Find the tangent plane of the graph of $f(x, y)$ at the point $(1, 1, 0)$. Write your answer in $ax + by + cz = d$ form.

$$\begin{aligned} f_x(1,1) &= \ln(x+y) + \frac{x-y}{x+y} \Big|_{(1,1)} = \ln(2) \\ f_y(1,1) &= -\ln(x+y) + \frac{x-y}{x+y} \Big|_{(1,1)} = -\ln(2) \end{aligned} \quad \left. \begin{array}{l} \\ \\ \end{array} \right\} \Rightarrow z = \ln(2)(x-1) - \ln(2)(y-1)$$

$$\Rightarrow \boxed{0 = \ln(2)x - \ln(2)y - z}$$

(b) (3 points) Find the rate of change of $f(x, y)$ at $(1, 1)$ toward the point $(4, 5)$.

$$\vec{u} = \frac{\langle 4, 5 \rangle - \langle 1, 1 \rangle}{\|\langle 3, 4 \rangle\|} = \frac{\langle 3, 4 \rangle}{5}$$

$$D_{\vec{u}} f(1,1) = \langle \ln(2), -\ln(2) \rangle \cdot \frac{\langle 3, 4 \rangle}{5} = \boxed{-\frac{\ln(2)}{5}}$$

(c) (2 points) What is the maximal directional derivative of $f(x, y)$ at $(1, 1)$? Show a little work.

$$\begin{aligned} \max D_{\vec{u}} f(1,1) &= \|\nabla f(1,1)\| = \|\langle \ln(2), -\ln(2) \rangle\| = \ln(2) \|\langle 1, -1 \rangle\| \\ &= \boxed{\ln(2) \sqrt{2}} \end{aligned}$$

2. (4 points) $G(A, W) = \frac{A}{A - W}$ is the specific gravity of a solid where A is the weight in air and W is the weight in water. If $A = 9 \pm 0.09$ pounds and $W = 5 \pm 0.03$ pounds, use differentials to estimate the maximum percentage error in the calculation of G .

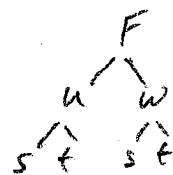
$$\max \% \text{ error} = \frac{\Delta G}{G} \cdot 100 \approx \frac{dG}{G} \cdot 100$$

$$100 \cdot \frac{dG}{G} = \frac{\left(\frac{(A-W) - A}{(A-W)^2} dA \right) + \left(\frac{A}{(A-W)^2} dW \right) \cdot 100}{\frac{A}{A-W}} = \left(\frac{-W}{A(A-W)} dA + \frac{dW}{A-W} \right) \cdot 100$$

$$\Rightarrow 100 \frac{dG}{G} = \left(\frac{-5}{9(4)} \cdot 0.09 + \frac{3}{4} \right) = \frac{5}{4} + \frac{3}{4} = \boxed{2\%} \text{ max error.}$$

Q4

3. (6 points) Let $F = F(u(s, t), w(s, t))$ where F , u , and w are differentiable functions with outputs given below. Find $\frac{\partial F}{\partial t}$ at $s = 1$ and $t = 0$. $F(1, 0) = 5$, $u(1, 0) = 2$, and $w(1, 0) = 3$.
 $\nabla u(1, 0) = \langle -2, 6 \rangle$, $\nabla w(1, 0) = \langle 5, 4 \rangle$, $\nabla F(1, 0) = \langle 5, 7 \rangle$, and $\nabla F(2, 3) = \langle -1, 10 \rangle$



$$\begin{aligned} \left. \frac{\partial F}{\partial t} \right|_{\substack{s=1 \\ t=0}} &= \left. \frac{\partial F}{\partial u} \cdot \frac{\partial u}{\partial t} + \frac{\partial F}{\partial w} \cdot \frac{\partial w}{\partial t} \right|_{\substack{s=1 \\ t=0 \\ u=2 \\ w=3}} \\ &= F_u(2, 3) \cdot u_t(1, 0) + F_w(2, 3) \cdot w_t(1, 0) \\ &= (-1) \cdot (6) + 10 \cdot 4 \\ &= \boxed{34} \end{aligned}$$

4. (6 points) $f(x, y, z) = 2x + 3y - z + \sin(\pi xyz)$. What is the equation for the tangent plane to the level surface of $f(x, y, z) = 4$ at $Q = (1, 1, 1)$?

$$\vec{n} = \nabla f(1, 1, 1) = \langle 2 + \pi y z \cos(\pi xyz), 3 + \pi x z \cos(\pi xyz), -1 + \pi xy \cos(\pi xyz) \rangle$$

$$\vec{n} = \langle 2 + \pi(-1), 3 + \pi(-1), -1 - \pi \rangle$$

\therefore the equation of the tangent plane is

$$\boxed{(2 - \pi)x + (3 - \pi)y - (1 + \pi)z = 4 - 3\pi}$$

Since

$$\langle 2 - \pi, 3 - \pi, -1 - \pi \rangle \cdot \langle 1, 1, 1 \rangle = (2 + 3 - 1) - \pi - \pi - \pi = 4 - 3\pi$$