Quiz 4 25 points Math 200 Name: ______________________

1. Let $f(x, y) = (x - y)\ln(x + y)$. You do not need to repeat work.

   (a) (4 points) Find the tangent plane of the graph of $f(x, y)$ at the point $(1, 1, 0)$. Write your answer in $ax + by + cz = d$ form.

$$f_x(1,1) = \ln(x+y) + \frac{x-y}{x+y}\Big|_{(1,1)} = \ln(2)$$

$$f_y(1,1) = -\ln(x+y) + \frac{x-y}{x+y}\Big|_{(1,1)} = -\ln(2)$$

$$\Rightarrow z = \ln(2)(x-1) - \ln(2)(y-1)$$

$$\Rightarrow \boxed{0 = \ln(2)x - \ln(2)y - z}$$

   (b) (3 points) Find the rate of change of $f(x, y)$ at $(1, 1)$ toward the point $(4, 5)$.

$$\hat{u} = \frac{\langle 4,5\rangle - \langle 1,1\rangle}{\|\langle 3,4\rangle\|} = \frac{\langle 3,4\rangle}{5}.$$

$$D_{\hat{u}} f(1,1) = \langle \ln(2), -\ln(2)\rangle \cdot \frac{\langle 3,4\rangle}{5} = \boxed{-\frac{\ln(2)}{5}}$$

   (c) (2 points) What is the maximal directional derivative of $f(x, y)$ at $(1, 1)$? Show a little work.

$$\max D_{\hat{u}} f(1,1) = \|\nabla f(1,1)\| = \|\langle \ln(2), -\ln(2)\rangle\| = \ln(2)\,\|\langle 1,-1\rangle\|$$
$$= \boxed{\ln(2)\sqrt{2}}$$

2. (4 points) $G(A, W) = \dfrac{A}{A - W}$ is the specific gravity of a solid where A is the weight in air and W is the weight in water. If $A = 9 \pm 0.09$ pounds and $W = 5 \pm 0.03$ pounds, use differentials to estimate the maximum percentage error in the calculation of G.

$$\max \% \text{ error} = \frac{\Delta G}{G} \cdot 100 \approx \frac{dG}{G} \cdot 100$$

$$100 \cdot \frac{dG}{G} = \frac{\left(\left|\frac{(A-W) - A}{(A-W)^2} dA\right| + \left|\frac{A}{(A-W)^2} dW\right|\right) \cdot 100}{\frac{A}{A-W}} = \left(\left|\frac{-W}{A(A-W)} dA\right| + \left|\frac{dW}{A-W}\right|\right) \cdot 100$$

$$\Rightarrow 100 \frac{dG}{G} = \left|\frac{-5}{9(4)}\right| \cdot 9 + \frac{3}{4} = \frac{5}{4} + \frac{3}{4} = \boxed{2\%} \text{ max error.}$$

Q4

3. (6 points) Let $F = F(u(s,t), w(s,t))$ where F, u, and w are differentiable functions with outputs given below. Find $\frac{\partial F}{\partial t}$ at $s=1$ and $t=0$. $F(1,0)=5$, $u(1,0)=2$, and $w(1,0)=3$.
$\nabla u(1,0) = \langle -2, 6\rangle$, $\nabla w(1,0) = \langle 5, 4\rangle$, $\nabla F(1,0) = \langle 5, 7\rangle$, and $\nabla F(2,3) = \langle -1, 10\rangle$

$$\left.\frac{\partial F}{\partial t}\right|_{\substack{s=1\\t=0}} = \left.\frac{\partial F}{\partial u}\cdot\frac{\partial u}{\partial t} + \frac{\partial F}{\partial w}\cdot\frac{\partial w}{\partial t}\right|_{\substack{s=1\\t=0\\u=2\\w=3}}$$

$$= F_u(2,3)\cdot u_t(1,0) + F_w(2,3)\cdot w_t(1,0)$$

$$= (-1)\cdot(6) + 10\cdot 4$$

$$= \boxed{34}$$

4. (6 points) $f(x,y,z) = 2x+3y-z+\sin(\pi xyz)$. What is the equation for the tangent plane to the level surface ~~of~~ $f(x,y,z) = 4$ at $Q = (1,1,1)$?

$$\vec{n} = \nabla f(1,1,1) = \langle 2+\pi yz\cos(\pi xyz),\ 3+\pi xz\cos(\pi xyz),\ -1+\pi xy\cos(\pi xyz)\rangle\Big|_{(1,1,1)}$$

$$\vec{n} = \langle 2+\pi(-1),\ 3+\pi(-1),\ -1-\pi\rangle.$$

$\therefore$ the equation of the tangent plane is

$$\boxed{(2-\pi)x+(3-\pi)y-(1+\pi)z = 4-3\pi}$$

Since

$$\langle 2-\pi, 3-\pi, -1-\pi\rangle\cdot\langle 1,1,1\rangle = (2+3-1)-\pi-\pi-\pi = 4-3\pi.$$