Quiz 6 25 points Math 200 Name: ______________________________

1. (7 points) Use polar coordinates to evaluate $I = \iint_R e^{-x^2-y^2}\,dA$ if R is the unit disk $x^2+y^2 \le 1$.

$$I = \int_0^{2\pi}\int_0^1 e^{-r^2}\cdot r\,dr\,d\theta$$

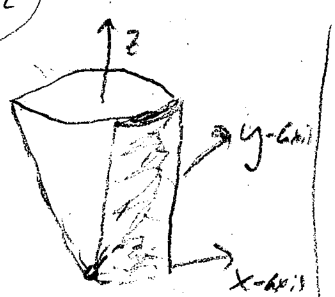
$$= 2\pi \cdot \left(\frac{e^{-r^2}}{-2}\right|_0^1$$

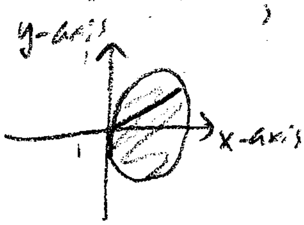
$$= -\pi\left(e^{-1}-1\right)$$

$$= \boxed{\pi\left(1-e^{-1}\right)}$$

2. (6 points) Use cylindrical coordinates to find the volume of the solid bounded above by the cone $z^2 = x^2+y^2$, below by the xy-plane, and on the sides by the cylinder $x^2+y^2 = 2x$.

$z = \sqrt{x^2+y^2}$





$$-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$$

$$0 \le r \le 2\cos\theta$$

$$\text{Vol} = \iint_R \sqrt{x^2+y^2}\,dV$$

$$\boxed{\begin{aligned} r^2 &= 2r\cos\theta \\ r &= 2\cos\theta \end{aligned}}$$

$$= \int_{-\pi/2}^{\pi/2}\int_0^{2\cos\theta} r\cdot r\,dr\,d\theta$$

$$= \int_{-\pi/2}^{\pi/2} \frac{r^3}{3}\Big|_0^{2\cos\theta}\,d\theta$$

$$= \frac{8}{3}\cdot 2\int_0^{\pi/2} \cos^3\theta\,d\theta$$

$$= \frac{16}{3}\int_0^{\pi/2} \cos\theta - \sin^2\theta\cos\theta\,d\theta$$

$$= \frac{16}{3}\left(\sin\theta - \frac{\sin^3\theta}{3}\right|_0^{\pi/2}$$

$$= \frac{16}{3}\cdot\left(1-\frac{1}{3}\right)$$

$$= \boxed{\frac{32}{9}}$$

Q6

3. (6 points) Use spherical coordinates to find the mass of the solid ball of radius 1 if the density is $\delta(\rho, \theta, \phi) = \frac{1}{1+\rho^2}$. Hint: $\frac{\rho^2}{1+\rho^2} = 1 - \frac{1}{1+\rho^2}$.

$$\text{Mass} = \int_0^{2\pi}\int_0^{\pi}\int_0^1 \frac{1}{1+\rho^2}\cdot\rho^2\sin\phi\,d\rho\,d\phi\,d\theta$$

$$= \int_0^{2\pi} d\theta \cdot \int_0^{\pi}\sin\phi\,d\phi \cdot \int_0^1 1-\frac{1}{1+\rho^2}\,d\rho$$

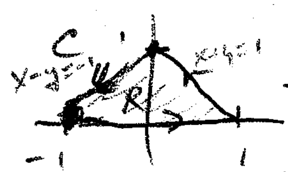
$$= 2\pi \cdot \left(-\cos\phi\Big|_0^{\pi}\right)\cdot\left(\rho - \arctan(\rho)\Big|_0^1\right.$$

$$= 2\pi\cdot(2)\cdot\left(1-\frac{\pi}{4}\right)$$

$$= \boxed{4\pi - \pi^2}$$

✓

4. (6 points) Use Green's Theorem and a double integral to evaluate the work done by the force $\vec{F} = \langle e^y, \sin(\pi x)\rangle$ in moving a particle once around a triangle starting at $(-1, 0)$ moving to $(1, 0)$ and then to $(0, 1)$ before returning to $(-1, 0)$.



C is C.C.W ✓

$$\Rightarrow \int_C e^y dx + \sin(\pi x)\,dy \overset{G.T.}{=} \iint_R \pi\cos(x) - e^y\,dA$$

$$= \int_0^1\int_{y-1}^{1-y} \underbrace{\pi\cos(x) - e^y}_{\text{even in } x}\,dx\,dy$$

$$= \int_0^1 2\left[\pi\sin(x) - e^y x\Big|_0^{1-y} dy\right.$$

$$= 2\int_0^1 \pi\sin(1-y) - e^y(1-y)\,dy$$

+	$y-1$	e^y
−	1	e^y
	0	e^y

$$= 2\left[\pi\cos(1-y) + e^y(y-1) - e^y\Big|_0^1\right.$$

$$= 2\left[(\pi - e^1) - (\pi\cos(1) - 2)\right] = \boxed{4 + 2\pi - 2e - 2\pi\cos(1)}$$