Quiz 8 25 points Math 200 Name: ______________________

1. use either Stokes' theorem exactly once or the divergence theorem exactly once to evaluate the following two surface integrals.

(a) (7 points) $I = \iint\limits_S \langle x^3, y^3, x^2y^2\rangle \cdot d\vec{S}$ and S is the boundary of the solid cylinder $x^2 + y^2 \leq 1$ between the planes $z = 0$ and $z = 2$ and is oriented out.

$$I \underset{\substack{\text{DIV}\\ \text{THM}}}{=} \iiint\limits_E 3x^2 + 3y^2 + 0 \, dV$$

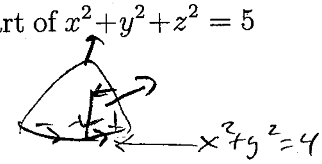
$$= 3\int_0^{2\pi}\int_0^1\int_0^2 r^2 \cdot r \, dz\, dr\, d\theta$$

$$= 3 \cdot 2\pi \cdot \int_0^1 r^3 dr \cdot \int_0^2 dz$$

$$= 6\pi \cdot \frac{r^4}{4}\Big|_0^1 \cdot 2$$

$$= 3\pi$$

(b) (6 points) $I = \iint\limits_S \nabla \times \vec{F} \cdot d\vec{S}$ if $\vec{F}(x, y, z) = \langle x^2yz, yz^2, z^3e^{xy}\rangle$ and if S is the part of $x^2+y^2+z^2 = 5$ that lies above the plane $z = 1$ and is oriented up.



$$I \underset{\text{S.T.}}{=} \oint\limits_{Bd(S)} \vec{F} \cdot d\vec{s} \quad ; \quad Bd(S): \vec{z}(t) = \langle 2\cos(t), 2\sin(t), 1\rangle$$

$$0 \leq t \leq 2\pi$$

$$= \int_0^{2\pi} \langle 8\cos^2(t)\sin(t), 2\sin(t), e^{4\cos(t)\sin(t)}\rangle \cdot \langle -2\sin(t), 2\cos(t), 0\rangle \, dt$$

$$= \int_0^{2\pi} -16\cos^2(t)\sin^2(t) + 4\sin(t)\cos(t) \, dt$$

$$0; 2\pi$$

$$= \int_0^{2\pi} -4(\sin(2t))^2 + \cancel{2\sin(2t)} \, dt$$

$$0; 4\pi$$

$$= \int_0^{2\pi} -2(1 - \cancel{\cos(4t)}) \, dt = -4\pi$$

Q8

2. (6 points) Use the Divergence Theorem to find $I = \iint_S x^2+2y^2+6z^2\, dS$ if S is the sphere $x^2+y^2+z^2=4$ oriented out.

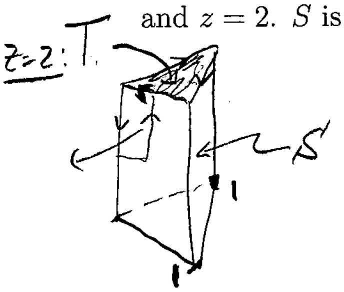
$$I = \iint_S 2\langle x, 2y, 6z\rangle \cdot \frac{\langle x,y,z\rangle}{2}\, dS$$

$$= 2\cdot \iint_S \langle x, 2y, 6z\rangle \cdot d\vec{S} \quad \text{(b/c } S \text{ is a sphere with } R=2.\text{)}$$

$$\underset{\text{D.T.}}{=} 2\cdot \iiint_{\rho\le 2} 1+2+6\, dV$$

$$= 18\cdot \mathrm{Vol}(\rho\le 2) = \cancel{18}^{\,6}\cdot \frac{4}{\cancel{3}}\pi\cdot 2^3 = 192\pi$$

3. (6 points) Use Stokes Theorem **twice** to compute $I = \iint_S \nabla\times\langle x+y, z^2-4, x\sqrt{y^2+1}\rangle\cdot d\vec{S}$ and S is the wedge-shaped box without a top that is bounded by the planes $x=0$, $y=0$, $z=0$, $x+y=1$, and $z=2$. S is oriented out.



$$I \overset{\text{S.T. (once)}}{=} \int_{Bd(S)} \langle x+y, z^2-4, x\sqrt{y^2+1}\rangle\cdot d\vec{s}$$

$$\langle \tfrac{\partial}{\partial x}, \tfrac{\partial}{\partial y}, \tfrac{\partial}{\partial z}\rangle \times \langle x+y, z^2-4, x\sqrt{y^2+1}\rangle$$
$$\langle \frac{xy}{\sqrt{y^2+1}}, \sqrt{y^2+1}, 0-1\rangle$$

T: z=2

is normal direction using right-hand-rule.
$d\vec{S} = -\hat{k}\,dx\,dy$

$$= \iint_T \langle \frac{xy}{\sqrt{y^2+1}}, \sqrt{y^2+1}, -1\rangle\cdot\langle 0,0,-1\rangle\, dx\,dy$$

$$= \iint_T 1\, dx\,dy$$

$$= \text{area}(T)$$

$$= \frac{1}{2}\cdot 1\cdot 1$$

$$= \boxed{\frac{1}{2}}$$