Quiz 9 25 points Math 200 Name: ______________________________

1. (7 points) Find and classify all of the critical points of $f(x,y) = (x^2 + y)e^{y/2}$ as either global or local minimums, maximums, or as a saddle point.

$\lim_{\substack{y \to \infty \\ x=0}} f(x,y) = \infty \Rightarrow$ no global max.

$f(x,y)$ is bounded below, so global mins might exist.

$f_x = 2xe^{y/2} = 0$ if $\underline{x=0}$.

$f_y = e^{y/2} + \frac{1}{2}(x^2+y)e^{y/2} = 0$

$\Rightarrow e^{y/2}\left[1 + \frac{x^2+y}{2}\right] = 0.$

If $x=0$, then $1 + \frac{y}{2} = 0$

$\Rightarrow \underline{y = -2.}$

$f_{xx} = 2e^{y/2}\Big|_{(0,-2)} = 2e^{-1}$

$f_{xy} = xe^{y/2} = f_{yx}\Big|_{(0,-2)} = 0$

$f_{yy} = \frac{1}{2}e^{y/2}\left[1 + \frac{x^2+y}{2}\right] + e^{y/2}\left[\frac{1}{2}\right]\Big|_{(0,-2)}$

$= \frac{1}{2}e^{-1}[0] + \frac{e^{-1}}{2}.$

$\therefore H(0,-2) = \begin{vmatrix} 2e^{-1} & 0 \\ 0 & \frac{1}{2}e^{-1} \end{vmatrix} = e^{-2} > 0.$

$f_{xx} > 0 \Rightarrow f(0,-2)$ is a minimum.

$f(0,-2) = -2e^{-1}$,

$\boxed{(0,-2) \text{ is a global minimum}}$

2. (6 points) Use Lagrange Multipliers to find the constrained global extreme values of $f(x,y) = \frac{1}{x} + \frac{1}{y}$ if the constraint is $\frac{1}{x^2} + \frac{1}{y^2} = 1$.

$C(x,y) = \frac{1}{x^2} + \frac{1}{y^2}$, so

$\nabla f = \lambda \nabla C$

$\Rightarrow \left\langle -\frac{1}{x^2}, -\frac{1}{y^2} \right\rangle = \lambda \left\langle \frac{-2}{x^3}, \frac{-2}{y^3} \right\rangle.$ $\lambda \neq 0$ b/c $-\frac{1}{x^2} < 0$ for all x.

Then $\left.\begin{aligned} -\frac{1}{x^2} &= \lambda\left(\frac{-2}{x^3}\right) \\ -\frac{1}{y^2} &= \lambda\left(\frac{-2}{y^3}\right) \end{aligned}\right\} \overset{\div}{\Rightarrow} \frac{y^2}{x^2} = \frac{y^3}{x^3} \Rightarrow x = y.$

So $\frac{1}{x^2} + \frac{1}{y^2} = 1 \Rightarrow \frac{1}{x^2} + \frac{1}{x^2} = 1$

$\Rightarrow x^2 = 2 \Rightarrow \underline{x = \pm\sqrt{2}}.$

$\Rightarrow (\sqrt{2},\sqrt{2}), (-\sqrt{2},-\sqrt{2})$ are the c.c.p.

$f(\sqrt{2},\sqrt{2}) = \frac{2}{\sqrt{2}} = \sqrt{2}$

$f(-\sqrt{2},-\sqrt{2}) = -\sqrt{2}$, so

$\boxed{\sqrt{2} \text{ is a constrained global max}}$

and

$\boxed{-\sqrt{2} \text{ is a constrained global min.}}$

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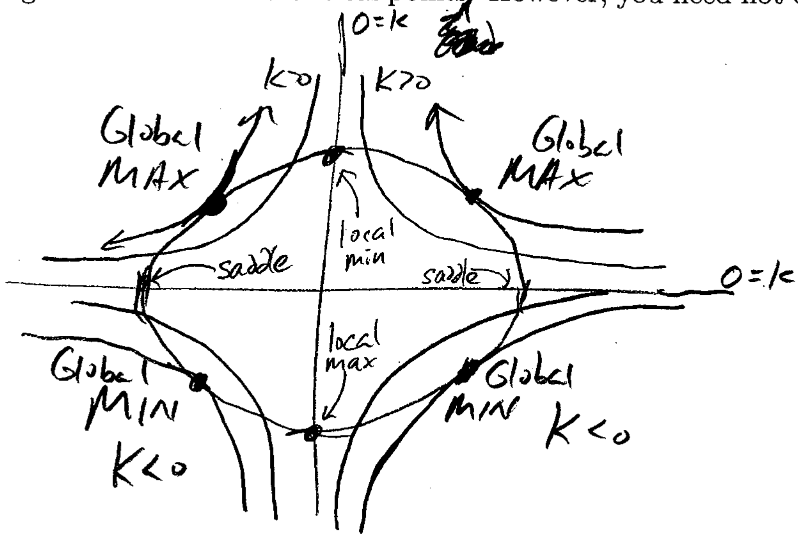
3. (6 points) Draw the contour plot and constraint curve corresponding to the Lagrange method for finding the constrained extreme values of $f(x, y) = x^2y$ with constraint $x^2 + y^2 = 1$. On your diagram label all global or local extreme values corresponding to the constrained critical points. However, you need not ~~do any algebra or~~ solve any system!

$k = x^2y$

$\Rightarrow y = \frac{k}{x^2}$ are the level curves.



4. (6 points) Suppose $P = (0, 2)$ is a critical point of the function $f(x, y)$ that has continuous second partials. Use the second partials test to classify P as a minimum, maximum, or saddle point if possible, or write "~~not possible~~." "DNA" for does not apply. Show work to defend your answer.

(a) (2 points) $f_{xx}(0, 2) = -1$, $f_{xy}(0, 2) = 6$, and $f_{yy}(0, 2) = 1$.

$H(0,2) = \begin{vmatrix} -1 & 6 \\ 6 & 1 \end{vmatrix} = -37 < 0 \Rightarrow (0,2)$ is a $\boxed{\text{saddle point}}$

(b) (2 points) $f_{xx}(0, 2) = -1$, $f_{xy}(0, 2) = 2$, and $f_{yy}(0, 2) = -8$.

$H(0,2) = \begin{vmatrix} -1 & 2 \\ 2 & -8 \end{vmatrix} = 4 > 0$; $f_{xx}(0,2) < 0 \Rightarrow (0,2)$ is a $\boxed{\text{maximum}}$

(c) (2 points) $f_{xx}(0, 2) = \cancel{-4}\ 4$, $f_{xy}(0, 2) = 6$, and $f_{yy}(0, 2) = 9$.

$H(0,2) = \begin{vmatrix} 4 & 6 \\ 6 & 9 \end{vmatrix} = 0 \Rightarrow \boxed{\text{DNA}}$