Test 1 100 points Math 200 Name: ______________________

1. (6 points) Find the equation of the plane in standard form that contains the point $P = (1, -1, 2)$, and is parallel to the vectors $\langle 4, 1, -1 \rangle$ and $\langle 1, 0, 2 \rangle$.

2. Let $f(x, y) = \ln(xy) + e^{xy^2} + 6$. You may use work from one part in another.

   (a) (6 points) Find the gradient of $f(x, y)$ at $(1, 1)$; that is, find $\nabla f(1, 1)$.

   (b) (6 points) Find the second partial derivatives of $f(x, y)$.

3. Let $\vec{u} = \langle 1, 2, -1 \rangle$, $\vec{p} = \langle 1, -2, 4 \rangle$, and $\vec{w} = 2\hat{\imath} - \hat{\jmath} + 3\hat{k}$. You may use work from one part in other parts.

(a) (4 points) Find the area of the triangle determined by $\vec{u}$ and $\vec{p}$.

(b) (4 points) Find the flux of $\vec{w}$ through the parallelogram spanned by $\vec{u}$ and $\vec{p}$ and oriented from $\vec{u}$ to $\vec{p}$.

(c) (4 points) Find $\vec{p}_{\parallel \vec{w}}$.

(d) (4 points) Find $\vec{p}_{\perp \vec{w}}$.

4. Let $\vec{\alpha}(t) = \langle \sin(2t), \cos(2t), t \rangle$ for all parts of this problem. You do not need to repeat work.

(a) (8 points) Find the arc length of the trace of $\vec{\alpha}(t)$ from $t = 0$ to $t = \pi$.

(b) (6 points) Find the equation of the osculating plane for the trace of $\vec{\alpha}(t)$ at $t = \pi$. Write the answer in $ax + by + cz = d$ form.

(c) (4 points) Find the curvature for the trace of $\vec{\alpha}(t)$ at $t = \pi$. Simplify your final answer.

5. (6 points) Write a position function with input t seconds for a particle that moves once counterclockwise with respect to $\hat{\jmath}$ around a circle every 2 seconds if the circle is in the plane $y = 4$ with center at $(2, 4, 1)$ and radius 3.

6. (4 points) Sketch the part of the plane $x + 2y + 4z = 8$ in the first octant (x, y, and z are all non-negative) with labeled positively oriented xyz-axes.

7. (6 points) Find $\vec{f}\,'(1)$ if $\vec{f}(t) = \vec{p}(t) \times \left\langle 1, t, t^2 \right\rangle$, $\vec{p}(1) = \hat{\imath} - 2\hat{\jmath} + 2\hat{k}$, and $\left.\dfrac{d\vec{p}}{dt}\right|_{t=1} = \langle 0, 1, -1 \rangle$. Show work to defend your answer.

8. (10 points) Find the mass of the wire that lies on the curve $y = x^3$ if it starts at $(0, 0)$ and ends at $(1, 1)$ and if the density of the wire is $\delta(x, y) = \sqrt{1 + 9xy}$ grams/cm.

9. Find the limit if possible, or prove that the limit does not exist. Show organized work to defend your answer.

   (a) (4 points) $\displaystyle\lim_{(x,y)\to(1,-1)} \frac{1+y}{x^2+xy}$

   (b) (4 points) $\displaystyle\lim_{(x,y)\to(0,1)} \frac{\sin(\arctan(y))}{x^2+y^2}$

10. (9 points) Find the work done by the vector field $\vec{F}(x, y, z) = -y\hat{\imath} + z\hat{\jmath} + x\hat{k}$ on a particle that moves on the trace of $\vec{\alpha}(t) = \langle 2 + t^{-1}, t^3, t^2 \rangle$ for $0 < t \leq 1$.

11. (5 points) Sketch $y = z^2 + x^2$ with labeled positively oriented xyz-axes.