Test 2 100 points Math 200 Name: ____________________

1. (6 points) Sketch the region of integration for $I = \int_1^3 \int_0^{3-x} f(x,y)\,dy\,dx$ and then rewrite $I$ with the order of integration switched.

2. (7 points) Find the tangent plane in standard form for the graph of $xz + 2x^2y + y^2z^3 = 11$ at the point $Q = (2,1,1)$.

3. (5 points) Approximate $f(2.3, 4.9)$ using a differential if $f(2,5) = 1$ and $\nabla f(2,5) = \langle 3, 8\rangle$.

Use the FTCLI to evaluate the integrals on this page whenever possible. If not possible, show work indicating this, and then evaluate the integral using another method.

4. (9 points) $I = \displaystyle\int_C [y + \sin(4y)]\,dx + [x + 4x\cos(4y)]\,dy$ if $C$ is parameterized by $\vec{\alpha}(t) = \langle \sin(t), t \rangle$ for $\pi/2 \leq t \leq \pi$.

5. (9 points) $I = \displaystyle\int_C [y + \sin(4y)]\,dx + [x + x\cos(4y)]\,dy$ if $C$ is parameterized by $\vec{\alpha}(t) = \left\langle t^3, t \right\rangle$ for $-\pi/2 \leq t \leq \pi/2$.

6. (9 points) Evaluate $I = \int_0^2 \int_0^y \int_x^1 6xy \, dz \, dx \, dy$.

7. (5 points) What is the tangent plane to the surface $z = f(x, y)$ at the point $P = (4, -1, 2)$ if $f(4, -1) = 2$ and $\nabla f(4, -1) = \langle 1, -3 \rangle$? Write your final answer in $ax + by + cz = d$ form.

8. (5 points) Find the rate of change of $f(x, y, z) = x\sqrt{y^2 + z^2}$ at the point $P = (2, 3, 4)$ in the direction of $\langle 1, 0, 5 \rangle$.

9. (9 points) Let $f(x, y) = x\ln(x^2 + y^2)$, $x(u, w) = 3uw$, $y = y(u, w)$, $y(-1, 1) = 1$, and $y_u(-1, 1) = -2$. Find $\dfrac{df}{du}$ when $u = -1$ and $w = 1$. Show organized work.

10. (9 points) Find the moment of inertia about the origin, $I_0$, for the solid hemisphere $0 \leq z \leq \sqrt{1 - x^2 - y^2}$ if the density is $\delta(x, y, z) = 4\sqrt{x^2 + y^2 + z^2}$.

11. (7 points) Find the mass of the solid that has nonnegative $x$, $y$, and $z$ coordinates for which $z^2 \leq 4x$ and $x + y \leq 1$; and if its density is $\delta(x, y, z) = z$.

12. (7 points) Use Green's Theorem to find the area of the region $R$ bounded by the closed curve $C$ parameterized by $\vec{p}(t) = \langle \cos(t) - \sin(t), \sin(t) + \cos(t) \rangle$ for $0 \leq t \leq 2\pi$.

13. (6 points) Write $I = \int_{-1}^{1} \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} \int_{0}^{x^2+y^2} \frac{1}{\sqrt{x^2+y^2+z^2}}\, dz\, dy\, dx$ using cylindrical coordinates.

**Do not evaluate or simplify!**

14. (7 points) Use Green's Theorem and a double integral to evaluate $I = \int_C y\, dx + x^2\, dy$ if $C$ is the closed curve that follows the parabola $y = x^2$ from $(-1, 1)$ to $(1, 1)$ and then returns on the straight line from $(1, 1)$ to $(-1, 1)$.