Test 3 100 points Math 200 Name: ____________________

1. (9 points) Find the equation of the tangent plane to the surface parameterized by $\vec{p}(u,w) = \left\langle u^2 - w^2, 2u + w, 4\sqrt{w} \right\rangle$ at the point where $u = 1$ and $w = 4$.

2. (8 points) Find the mass of the piece of the paraboloid $z = x^2 + y^2$ that lies between $z = 1$ and $z = 2$ if density is $\delta(x,y,z) = \dfrac{z - x}{\sqrt{4x^2 + 4y^2 + 1}}$ g/cm^2.

3. (9 points) Use a surface integral to find the flux of $\vec{F}(x,y,z) = \langle x, x, xyz \rangle$ **out of** the piece of the cylinder $x^2 + y^2 = 1$ that is above the plane $z = 0$ and below the plane $z = x$.

4. (8 points) Find and classify all critical points for $f(x,y) = x^3 - 6xy + y^3$. A complete answer will include either the word "local" or "global."

5. (8 points) Evaluate $I = \iint_R \frac{x-2y}{3x-y}\, dA$ using a substitution if R is bounded by $x - 2y = 0$, $x - 2y = 2$, $3x - y = 1$, and $3x - y = 8$.

6. (9 points) Use Lagrange Multipliers to find the minimum value of $f(x, y) = x^2y$ if the inputs are constrained to lie on the ellipse $x^2 + 2y^2 = 6$. Sketch a contour plot with the significant level curves to verify your answer.

7. (8 points) Use Stokes' Theorem **once** to find $I = \iint\limits_S \nabla \times \vec{F} \cdot d\vec{S}$ if $\vec{F}(x, y, z) = \langle y, xyz, y^2z^2 \rangle$ and S is the piece of the cone $z^2 = x^2 + y^2$ between $z = 0$ and $z = 3$ oriented down.
Hint: you must evaluate a line integral.

8. (9 points) Use a surface integral to find the flux of $\vec{F}(x, y, z) = \langle y, -x, z \rangle$ through the piece of the sphere $x^2 + y^2 + z^2 = 4$ that lies above $z = -1$ if it is oriented out.

9. (9 points) Use Stokes Theorem to find the work done by $\vec{F}(x, y, z) = \langle z, 2x, 4y \rangle$ on a particle that moves around the triangle from vertex $(3, 0, 0)$ to vertex $(0, 3, 0)$ to vertex $(0, 0, 6)$ before returning to $(3, 0, 0)$. Hint: You must use a double integral.

10. (9 points) Use the Divergence Theorem **once** to find the flux of $\vec{F}(x, y, z) = \left\langle 2xy^2, z^2, -2y \right\rangle$ out of the boundary of the solid that lies above $z = 0$, below $z = x^2 + y^2$ and inside the cylinder $x^2 + y^2 = 1$. Hint: you must evaluate a triple integral.

11. (8 points) Use the Divergence Theorem to find $I = \iint\limits_{Bd(E)} x^2 + 2y^2 - 4z\, dS$ if E is the solid ball $x^2 + y^2 + z^2 \leq 4$.

12. (6 points) The velocity field of a fluid $\vec{v}$ g·cm/sec has a divergence at the point $P = (2, 2, 2)$ of 3 g/sec. Estimate the flow rate (or flux) out of the sphere of radius 0.1 cm centered at P.