Quiz 2 25 points Math 200 Name: ____________________

1. Let $\vec{\alpha}(t) = \left\langle t, \frac{4t^{3/2}}{3}, t^2 \right\rangle$ for all parts of this problem. You do not need to repeat work.

(a) (5 points) Find the arc length of the trace of $\vec{\alpha}(t)$ from $t = 0$ to $t = 4$.

$\vec{\alpha}'(t) = \langle 1, 2t^{1/2}, 2t \rangle$; $s = \int_C ds$

$\Rightarrow s = \int_0^4 \sqrt{1+4t+4t^2}\, dt$

$= \int_0^4 \sqrt{(1+2t)^2}\, dt$

$= \int_0^4 (1+2t)\, dt$

$\Rightarrow s = t + t^2 \Big|_0^4 = \boxed{20}$

(b) (4 points) Find the equation of the osculating plane for the trace of $\vec{\alpha}(t)$ at $t = 1$.

$\alpha(1) = \langle 1, \frac{4}{3}, 1 \rangle$ is a point in the plane. $\vec{v} \times \vec{a}$ is $\perp$ to plane.

$\vec{\alpha}'(1) = \langle 1, 2, 2 \rangle = \vec{v}$

$\times\ \vec{\alpha}''(1) = \langle 0, 1, 2 \rangle = \vec{a}$

$\vec{n} = \langle 2, -2, 1 \rangle$.

$\therefore\ 2x - 2y + z = \langle 2, -2, 1 \rangle \cdot \langle 1, \frac{4}{3}, 1 \rangle$

$\Rightarrow 2x - 2y + z = 2 - \frac{8}{3} + 1$

$\Rightarrow \boxed{2x - 2y + z = \frac{1}{3}}$ OR $\boxed{6x - 6y + 3z = 1}$

(c) (4 points) Find the curvature for the trace of $\vec{\alpha}(t)$ at $t = 1$.

$\kappa(1) = \frac{\|\vec{v}(1) \times \vec{a}(1)\|}{v^3(1)} = \frac{\|\langle 2, -2, 1 \rangle\|}{\left(\sqrt{1+4+4}\right)^3} = \frac{3}{27} = \boxed{\frac{1}{9}}$

2. (4 points) Find and simplify $I = \int_{-2}^{2} \langle \underbrace{\cancel{(t^3 - t)\sin^2(t)}}_{\text{ODD}}^{\to 0}, \cos^2(\pi t), \sin(2\pi t)\sin(3\pi t)\rangle\, dt$. Show work to defend your answer.

$$\Rightarrow I = \left\langle 0,\ \int_{-2}^{2} \frac{1+\overset{\to 0;\,4T}{\cancel{\cos(2\pi t)}}}{2}\, dt,\ \int_{-2}^{2} \frac{\overset{\to 0;\,2T}{\cancel{\cos(\pi t)}} - \overset{\to 0;\,10T}{\cancel{\cos(5\pi t)}}}{2}\, dt \right\rangle$$

$$\Rightarrow \boxed{I = \langle 0, 2, 0\rangle}$$

3. (4 points) Find $f'(1)$ if $f(t) = \vec{p}(t) \cdot \langle e^t, \ln(t), \arctan(t)\rangle$, $\vec{p}(1) = \langle -e, -1, 3\rangle$, and $\left.\frac{d\vec{p}}{dt}\right|_{t=1} = \langle 1, 3, 4\rangle$. Show work to defend your answer.

$$f'(1) = \vec{p}\,'(1) \cdot \langle e, 0, \tfrac{\pi}{4}\rangle + \vec{p}(1) \cdot \langle e, \tfrac{1}{1}, \tfrac{1}{2}\rangle$$

$$= \langle 1,3,4\rangle \cdot \langle e, 0, \tfrac{\pi}{4}\rangle + \langle -e, -1, 3\rangle \cdot \langle e, 1, \tfrac{1}{2}\rangle$$

$$= e + \pi - e^2 - 1 + \tfrac{3}{2}$$

$$= \boxed{\tfrac{1}{2} + \pi + e - e^2}$$

4. (a) (2 points) Convert $P(-3, 4, \sqrt{11})$ to cylindrical coordinates.

$r = \sqrt{9+16} = 5$

$\tan\theta = -\frac{4}{3}$, θ in Quad II

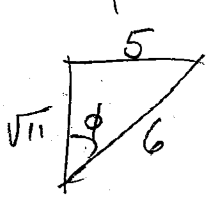
$\Rightarrow \theta = \tan^{-1}(-\frac{4}{3}) + \pi$

$$\Rightarrow \boxed{P = \left(5, \pi + \tan^{-1}\left(\tfrac{-4}{3}\right), \sqrt{11}\right)_C}$$

Note: $\pi + \tan^{-1}(\frac{-4}{3}) = \pi - \tan^{-1}(\frac{4}{3})$

(b) (2 points) Convert $P(-3, 4, \sqrt{11})$ to spherical coordinates.

$\rho^2 = 9 + 16 + 11 = 36 \Rightarrow \rho = 6$



$\Rightarrow \cos\phi = \frac{\sqrt{11}}{6}$

$$\Rightarrow \boxed{P = \left(6, \pi + \tan^{-1}\left(\tfrac{-4}{3}\right), \cos^{-1}\left(\tfrac{\sqrt{11}}{6}\right)\right)_S}$$