Quiz 3 25 points Math 200 Name: ______________________

1. Let $f(x,y,z) = x^2y - e^{yz} + 4$. You do not need to repeat work.

(a) (3 points) Find the gradient of $f(x,y,z)$; that is, find ∇f.

$$\nabla f = \langle 2xy, x^2 - ze^{yz}, -ye^{yz} \rangle$$

(b) (2 points) Find and simplify $f_{yy}(x,y,z)$.

$$f_y = x^2 - ze^{yz} \Rightarrow \boxed{f_{yy} = -z^2 e^{yz}}$$

(c) (2 points) Find and simplify $\dfrac{\partial^2 f}{\partial z \partial y}$.

$$f_y = x^2 - ze^{yz} \Rightarrow f_{yz} = -yze^{yz} - e^{yz} = \boxed{-e^{yz}(yz+1)}$$

2. Find the limit if possible, or prove that the limit does not exist. Show organized work to defend your answer.

(a) (3 points) $\displaystyle\lim_{(x,y)\to(0,0)} \left(\frac{3x^2 + x - y^2}{x^2 - 2x + y^2}\right)$

$$x = 0 \Rightarrow \lim_{y\to 0^+} \frac{-y^2}{y^2} = -1$$

$$y = 0 \Rightarrow \lim_{x\to 0^+} \frac{3x^2+x}{x^2-2x} = \lim_{x\to 0^+} \frac{3x+1}{x-2} = -\frac{1}{2}$$

$-1 \neq -\frac{1}{2}$
$\Rightarrow$ the limit DNE
by the path theorem,

(b) (3 points) $\displaystyle\lim_{(x,y)\to(1,0)} \frac{y}{\tan^{-1}\left(\frac{y}{x}\right)} = \lim_{\substack{r\to 1\\ \theta\to 0}} \frac{r\sin\theta}{\theta} = \lim_{r\to 1} r \cdot \lim_{\theta\to 0} \frac{\sin\theta}{\theta}$

$$= 1 \cdot 1 = \boxed{1}$$

(Note: your quiz had $\tan\left(\frac{y}{x}\right)$, not $\tan^{-1}\left(\frac{y}{x}\right)$. Converting to polar $\Rightarrow \displaystyle\lim_{\substack{r\to 1\\ \theta\to 0}} \frac{r\sin\theta}{\tan(\tan\theta)}$

$$= \lim_{r\to 1} r \cdot \lim_{\theta\to 0} \frac{\sin\theta}{\tan(\tan\theta)} \overset{L'H}{=} 1 \cdot \lim_{\theta\to 0} \frac{\cos\theta}{\sec^2(\tan\theta)\cdot\sec^2\theta} = 1 \cdot \frac{1}{1\cdot 1} = \boxed{1}$$ as well

3. (6 points) Set-up and evaluate an integral that represents the mass of the elliptical wire lying on $9x^2 + 4y^2 = 36$ if its density is $\delta(x, y) = \sqrt{\frac{9}{4}x^2 + \frac{4}{9}y^2}$ grams per centimeter.

$\vec{\alpha}(t) = \langle 2\cos(t), 3\sin(t) \rangle, \quad 0 \le t \le 2\pi$

$\Rightarrow \vec{\alpha}'(t) = \langle -2\sin(t), 3\cos(t) \rangle$

$\therefore$ Mass $= \int_C \delta \, ds$

$= \int_0^{2\pi} \sqrt{9\cos^2(t) + 4\sin^2(t)} \sqrt{4\sin^2(t) + 9\cos^2(t)} \, dt$

$= \int_0^{2\pi} 9\cos^2(t) + 4\sin^2(t) \, dt$

$= \frac{1}{2} \int_0^{2\pi} 9 + \cancel{9\cos(2t)} + 4 - \cancel{4\cos(2t)} \, dt = \boxed{13\pi}$ $\left(\frac{1}{2} \cdot 13 \cdot 2\pi\right)$

$\to 0 ; 2T \qquad \to 0 ; 2T$

4. (6 points) Evaluate $I = \int_C 5xyz^2 \, dx + (x^3 - 5) \, dy + 5xyz \, dz$ if C is the trace of $\vec{\alpha}(t) = \langle 1, t, t^2 \rangle$ for $0 \le t \le 1$.

$\vec{\alpha}'(t) = \langle 0, 1, 2t \rangle$

$\Rightarrow \begin{cases} dx = 0 \\ dy = 1\,dt \\ dz = 2t\,dt \end{cases}$

$\Rightarrow I = \int_0^1 0 + (1^3 - 5) + 5(1)(t)(t^2) \cdot 2t \, dt$

$= -4 + \left(\frac{10t^5}{5}\right|_0^1 = \boxed{-2}$