Quiz 1 25 points Math 200 Name: ______________________

1. (4 points) Find the equation of the plane in standard form that contains the points $P = \left(\sqrt{2}, \frac{\pi}{4}, 1\right)_C$, $Q = (3, 2, 2)$, $R = (1, 2, 3)$. Hint: Convert the cylindrical point to rectangular coordinates.

$x = \sqrt{2}\cos(\pi/4) = \frac{2}{2} = 1$

$y = \sqrt{2}\sin(\pi/4) = 1$

$\boxed{\therefore P = (1,1,1).}$

$\vec{PQ} = \langle 2,1,1\rangle$
$\times\ \vec{PR} = \langle 0,1,2\rangle$
$\langle 1,-4,2\rangle$

Use $\vec{n} = \langle 1,-4,2\rangle$

$\vec{n}\cdot\vec{P} = -1$

Then the equation of the plane is

$$\boxed{x - 4y + 2z = -1}$$

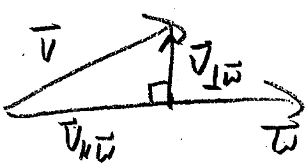
2. Let $\vec{v} = -2\hat{\imath} + 2\hat{\jmath} + \hat{k}$ and $\vec{w} = \langle 3, 0, 4\rangle$. You may use work from one part in other parts.

(a) (2 points) Find the cosine of the angle between $\vec{v}$ and $\vec{w}$.

$$\vec{v}\cdot\vec{w} = \|\vec{v}\|\,\|\vec{w}\|\cos\theta \Rightarrow \cos\theta = \frac{-6+0+4}{\sqrt{9+16}\,\sqrt{4+4+1}} \Rightarrow \boxed{\cos\theta = \frac{-2}{15}}$$

(b) (4 points) Find $\vec{v}_{\parallel\vec{w}}$ and $\vec{v}_{\perp\vec{w}}$.

$$\vec{v}_{\parallel\vec{w}} = \frac{\vec{v}\cdot\vec{w}}{\vec{w}\cdot\vec{w}}\vec{w} = \boxed{\frac{-2}{25}\langle 3,0,4\rangle}$$

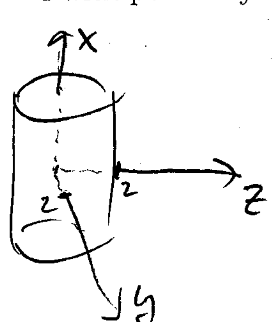


$$\Rightarrow \vec{v}_{\perp\vec{w}} = \vec{v} - \vec{v}_{\parallel\vec{w}} = \langle -2,2,1\rangle + \frac{\langle 6,0,8\rangle}{25} = \boxed{\frac{\langle -44,50,33\rangle}{25}}$$

(c) (2 points) Find the work done by a force $\vec{v}$ applied to a particle with displacement $\vec{w}$.

$$W = \vec{v}\cdot\vec{w} = \boxed{-2}$$ (units of work)

3. (1 point) Sketch $y^2 + z^2 = 4$ with positively oriented xyz-axes.



4. Let $\vec{u} = 2\hat{\imath} + 2\hat{\jmath} + \hat{k}$ and $\vec{p} =< 2, -2, 1 >$. You may use ~~answers~~ results from one part ~~for~~ in other parts. ✓

(a) (4 points) Find the area of the triangle spanned by $\vec{p}$ and $\vec{u}$

$$\begin{array}{rl} & \vec{u} = \langle 2,2,1\rangle \\ \times & \vec{p} = \langle 2,-2,1\rangle \\ \hline \vec{u}\times\vec{p} & = \langle 4,0,-8\rangle \end{array}$$

$\Rightarrow$ triangle area $= \frac{1}{2}\|4\langle 1,0,-2\rangle\| = \boxed{2\sqrt{5}}$

(length units^2)

(b) (2 points) Find the flux of the vector field $\vec{F} =< 1, -3, 2 >$ through the parallelogram spanned by $\vec{u}$ and $\vec{p}$ and oriented from $\vec{u}$ to $\vec{p}$.

$$\text{Flux} = \vec{F}\cdot(\vec{u}\times\vec{p}) = \langle 1,-3,2\rangle\cdot\langle 4,0,-8\rangle = 4-16 = \boxed{-12}$$

(c) (1 point) Find the volume of the box spanned by $\vec{F}$, $\vec{p}$, and $\vec{u}$.

$$|\text{flux}| = \boxed{12}$$

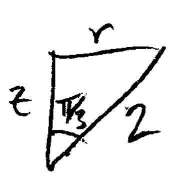
5. (3 points) Find a position function and the ~~parametric~~ coordinate ✓ equations for the line that contains the point $P = (2, 1, 1)$, and is perpendicular to the plane $x + 2y + 3z = 4$. $\Rightarrow$ direction $= \langle 1,2,3\rangle$.

$$\therefore\ \vec{p}(t) = \langle 2,1,1\rangle + t\langle 1,2,3\rangle \Rightarrow \boxed{\vec{p}(t) = \langle 2+t,\ 1+2t,\ 1+3t\rangle}$$

and the coordinate equations are:

$$\boxed{\begin{aligned} x &= 2+t \\ y &= 1+2t \\ z &= 1+3t \end{aligned}}$$

6. (2 points) Convert $Q = (\cancel{8}\,2, 0, \cancel{\sqrt{3}}\,\tfrac{\pi}{3})_S$ to cylindrical and rectangular coordinates. ✓



$$\Rightarrow \left.\begin{aligned} r &= 2\sin\left(\tfrac{\pi}{3}\right) = \sqrt{3} \\ z &= 2\cos\left(\tfrac{\pi}{3}\right) = 1 \\ x &= \sqrt{3}\cos(0) = \sqrt{3} \\ y &= \sqrt{3}\sin(0) = 0 \end{aligned}\right\} \Rightarrow \boxed{Q = (\sqrt{3},0,1)_c = (\sqrt{3},0,1)}$$