Quiz 3 25 points Math 200 Name: ______________________

1. Let $f(x, y, z) = \ln(x + yz)$. You do not need to repeat work.

(a) (3 points) Find the gradient of $f(x, y, z)$; that is, find ∇f.

$$\boxed{\nabla f = \left\langle \frac{1}{x+yz}, \frac{z}{x+yz}, \frac{y}{x+yz} \right\rangle}$$

(b) (2 points) Find and simplify $f_{yy}(x, y, z)$.

$$f_{yy} = \left(\frac{z}{x+yz}\right)_y = \boxed{\frac{-z^2}{(x+yz)^2}}$$

(c) (2 points) Find and simplify $\dfrac{\partial^2 f}{\partial z \partial x}$.

$$f_{zx} = \left(\frac{y}{x+yz}\right)_x = \boxed{\frac{-y}{(x+yz)^2}}$$

2. Find the limit if possible, or prove that the limit does not exist. Show organized work to defend your answer.

(a) (3 points) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{x^2 + 4y^2 + x^3}{x^2 + y^2}$

$$\left.\begin{aligned} &\lim_{\substack{x\to 0^+\\ y=0}} \frac{x^2+x^3}{x^2} = \lim_{x\to 0^+} 1+x = 1 \\ &\lim_{\substack{y\to 0^+\\ x=0}} \frac{4y^2}{y^2} = 4 \end{aligned}\right\} \quad 1 \neq 4 \Rightarrow \boxed{\text{limit DNE}}$$

(b) (3 points) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{3xy^2}{x^2 + y^2} = \lim_{\substack{r\to 0^+\\ \theta \text{ free}}} \frac{3r\cos\theta(r\sin\theta)^2}{r^2} = \lim_{\substack{r\to 0^+\\ \theta \text{ free}}} 3r\cos\theta\sin\theta = \boxed{0}$

because $-3r \leq 3r\cos\theta\sin\theta \leq 3r$ and $\lim_{r\to 0^+} \pm 3r = 0$, so the squeeze theorem $\Rightarrow$ limit is 0.

$y=-\sqrt{4-x^2}$ (the lower ~~the lower half of~~ half circle)

3. (6 points) Set up an integral that represents the mass of the wire that lies on ~~$x = \sin(y)$ from $(0,0)$ to $(0,2\pi)$~~ in the xy-plane if the density of the wire is $\delta(x,y) = \cancel{1+y}$ $8+x^2y$, and then **evaluate** the integral.

~~$x^2+y^2=1$~~

$$\text{Mass} = \int_C \delta\, ds \quad ; \quad C: \vec{\alpha}(t) = \langle 2\cos t, 2\sin t \rangle, \quad \pi \le t \le 2\pi$$

$$\vec{\alpha}'(t) = \langle -2\sin t, 2\cos t \rangle$$

$$\text{Mass} = \int_\pi^{2\pi} (8+8\cos^2(t)\sin(t))\sqrt{4(\sin^2 t+\cos^2 t)}\, dt$$

$$= 16\cdot\pi - \frac{16\cos^3(t)}{3}\Big|_\pi^{2\pi} = 16\pi - \frac{16}{3}(1-(-1)) = \boxed{16\pi - \frac{32}{3}}$$

mass units

↑ ↑ Room

4. (6 points) Evaluate $I = \int_C \sqrt[3]{z}\,dx + \sqrt[3]{x}\,dy + \cancel{\sin^2(y)}\, e^y\, dz$ if C is the curve from $(-1,-1,-1)$ to $(1,1,1)$ on the trace of $\vec{\alpha}(t) = \langle t^3, t, t^3\rangle$. If I = work on particle, what is the force field?

$$\vec{\alpha}'(t) = \langle 3t^2, 1, 3t^2\rangle \Rightarrow I = \int_{-1}^{1} \underbrace{t\cdot 3t^2}_{\text{ODD}} + \underbrace{t\cdot 1}_{\text{ODD}} + e^t\cdot 3t^2\, dt$$

$$\Rightarrow I = 3\int_{-1}^{1} t^2 e^t\, dt = 3\left[t^2e^t - 2te^t + 2e^t\right]_{-1}^{1}$$

$\oplus$	t^2	e^t
$\ominus$	$2t$	e^t
$\oplus$	2	e^t
	0	e^t

$$= 3\left(\left[e(1-2+2) - e^{-1}(1+2+2)\right]\right)$$

$$= \boxed{3(e-5e^{-1})}$$

$$\boxed{\text{Force field is } \langle \sqrt[3]{z}, \sqrt[3]{x}, e^y\rangle = \vec{F}(x,y,z)}$$