Quiz 5 25 points Math 200 Name: ______________________

1. (7 points) Use the FTCLI to find $\int_C \vec{F} \cdot d\vec{s}$ if $\vec{F}(x,y,z) = \langle e^x y + 3, e^x + 2yz, y^2 \rangle$ and $C : \vec{\alpha}(t) = <$ new line
$1 + \cos(t), 1 + \sin(t), t >$, $0 \le t \le \pi$.

$\phi = \int \phi_x dx = e^x y + 3x + c(y,z)$

$e^x + 2yz = \phi_y = e^x + c_y \Rightarrow c_y = 2yz \Rightarrow c = y^2 z + c_1(z)$

$\therefore \phi = e^x y + 3x + y^2 z + c_1(z)$.

$y^2 = \phi_z = y^2 + (c_1)_z \Rightarrow c_1 = \text{constant}$.

$\therefore \phi = e^x y + 3x + y^2 z + \cancel{c}^{\to 0}$

OR let $\phi(0,0,0) = 0$. Then

$\phi(a,b,c) = \int_0^a 3\,dx + \int_0^b e^a dy + \int_0^c b^2 dz$

$= 3a + be^a + cb^2$

$\Rightarrow \phi(x,y,z) = 3x + ye^x + zy^2$

FTCLI $\Rightarrow \int_C \vec{F} \cdot d\vec{s} = \phi(\alpha(\pi)) - \phi(\alpha(0)) = \phi(0,1,\pi) - \phi(2,1,0)$

$= (e^0 + 0 + \pi) - (e^2 + 6 + 0) = \boxed{\pi - 5 - e^2}$

2. (6 points) Calculate $\int_{-1}^{1} \int_0^y \int_{-xy}^{xy} 5y + \cancel{\sin(z)}^{\to 0} \, dzdxdy$. odd

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$= \int_{-1}^{1} \int_0^y 5y(2xy)\, dx\, dy$

$= 5 \int_{-1}^{1} y^2 x^2 \Big|_0^y dy$

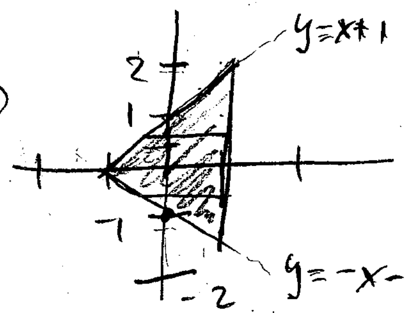
$= 5 \int_{-1}^{1} y^4 dy$

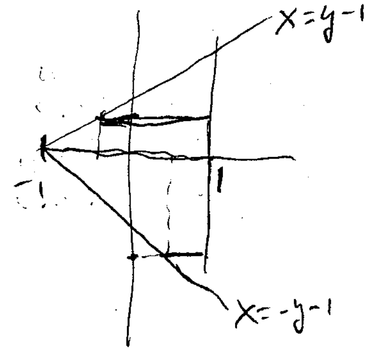
$= 10 \frac{y^5}{5}\Big|_0^1 = \boxed{2}$

3. (6 points) Sketch the region of integration for $I = \int_{-1}^{1}\int_{-x-1}^{x+1} f(x,y)\,dydx$ and then write I as an integral expression with the order of integration switched.

$-1 \le x \le 1$

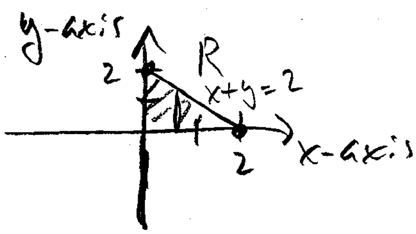
$-x-1 \le y \le x+1 \Rightarrow$





$$\boxed{\Rightarrow I = \int_{-2}^{0}\int_{-y-1}^{1} f(x,y)\,dx\,dy + \int_{0}^{2}\int_{y-1}^{1} f(x,y)\,dx\,dy}$$

4. (6 points) Use a double integral to find the volume of the solid in the first octant bounded by the coordinate planes and $x + y + 2z = 2$.



$$Vol = \int_0^2 \int_0^{2-x} 1 - \frac{x+y}{2}\,dy\,dx$$

$x+y+2z=2$

$\Rightarrow z = 1 - \frac{x+y}{2}$

$$= \text{Area } R - \int_0^2 \left.\frac{(x+y)^2}{4}\right|_0^{2-x} dx$$

$$= \left(\tfrac{1}{2}\cdot 2\cdot 2\right) - \int_0^2 \frac{2^2}{4} - \frac{x^2}{4}\,dx$$

$$= 2 - 2 + \left.\frac{x^3}{12}\right|_0^2 = \boxed{\frac{2}{3}}$$

Note: solid looks like a cone

So $Vol = \frac{1}{3} \overset{2}{B} H$

$= \frac{1}{3}\cdot 2\cdot 1$ ✓