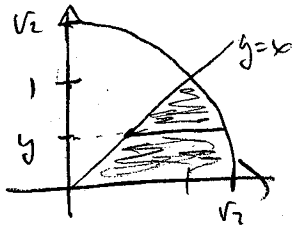
Quiz 6 25 points Math 200 Name: ______________________________

1. (5 points) Use polar coordinates to evaluate $I = \int_0^1 \int_y^{\sqrt{2-y^2}} x + y\,dx\,dy$.



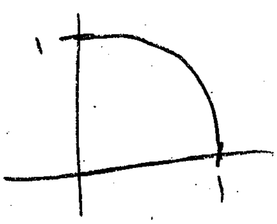
$$\Rightarrow I = \int_0^{\pi/4}\int_0^{\sqrt{2}} (r\cos\theta + r\sin\theta)\, r\, dr\, d\theta$$

$$= \int_0^{\pi/4} \cos\theta + \sin\theta\, d\theta \cdot \int_0^{\sqrt{2}} r^2\, dr$$

$$= \left(\sin\theta - \cos\theta\right|_0^{\pi/4} \cdot \left.\frac{r^3}{3}\right|_0^{\sqrt{2}}$$

$$= [0 - (-1)] \cdot \frac{2\sqrt{2}}{3} = \boxed{\frac{2\sqrt{2}}{3}}$$

2. (4 points) Find the center of mass for the quarter-disk $x^2 + y^2 \le 1$, $x \ge 0$ and $y \ge 0$ if the density is $\delta(x, y) = y$ grams per square meter.



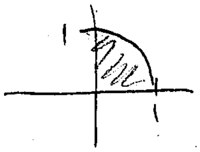
$$M = \int_0^{\pi/2}\int_0^1 r^2 \sin\theta\, dr\, d\theta = \left.\frac{r^3}{3}\right|_0^1 \cdot \left(-\cos\theta\right|_0^{\pi/2} = \frac{1}{3}$$

$$M_y = \int_0^{\pi/2}\int_0^1 r^3 \sin\theta\cos\theta\, dr\, d\theta = \left.\frac{r^4}{4}\right|_0^1 \cdot \left(\left.\frac{\sin^2\theta}{2}\right|_0^{\pi/2}\right) = \frac{1}{8}$$

$$M_x = \int_0^{\pi/2}\int_0^1 r^3 \sin^2\theta\, dr\, d\theta = \left.\frac{r^4}{4}\right|_0^1 \cdot \int_0^{\pi/2} \frac{1-\cos(2\theta)}{2}\, d\theta = \frac{\pi}{16}$$

$$\boxed{\therefore \text{ c.m.} = \left(\frac{3}{8}, \frac{3\pi}{16}\right)}$$

3. (4 points) Find the moment of inertia about the origin for the quarter-disk $x^2 + y^2 \le 1$, $x \ge 0$ and $y \ge 0$ if the density is $\delta(x, y) = y$ grams per square meter.



$$I_0 = \int_0^{\pi/2}\int_0^1 r^2 \cdot r\sin\theta \cdot r\, dr\, d\theta$$

$$= \int_0^{\pi/2} \sin\theta\, d\theta \cdot \int_0^1 r^4\, dr$$

$$= 1 \cdot \left.\frac{r^5}{5}\right|_0^1 = \frac{1}{5}$$

4. (6 points) Use spherical coordinates to evaluate $I = \iiint_E \frac{1}{x^2+y^2+z^2}\, dV$ if E is the solid that lies between the spheres $\rho = 1$ and $\rho = 5$.

$$I = \int_0^{2\pi}\int_0^{\pi}\int_1^{5} \frac{1}{\rho^2}\cdot \rho^2 \sin\phi \, d\rho\, d\phi\, d\theta$$

$$= \int_0^{2\pi} d\theta \cdot \int_0^{\pi} \sin\phi\, d\phi \cdot \int_1^5 d\rho$$

$$= 2\pi \cdot 2 \cdot 4$$

$$= 16\pi$$

5. (6 points) Use Green's Theorem **once** to evaluate $\int_C \vec{F} \cdot d\vec{s}$ if $\vec{F}(x,y) =< -y, x >$ and if C is the curve that starts at $(1,0)$ and rotates once around the ellipse $x^2 + \frac{y^2}{4} = 1$ in the counterclockwise direction.

$$\int_C F \cdot ds = \iint_{\text{ellipse}} \frac{\partial x}{\partial x} - \frac{\partial(-y)}{\partial y}\, dA = \iint_{\text{ellipse}} 1-(-1)\, dA$$

$$= 2\cdot(\text{Area of ellipse})$$

$$= 2 \cdot 2 \cdot 1\pi$$

$$= 4\pi$$