Quiz 9 25 points Math 200 Name: ______________________

1. (8 points) Find and classify all of the critical points of $f(x, y) = k^3xy + x^{-1} + y^{-1}$ for all possible values of k.

x^{-1} y^{-1}

① $f_x = k^3y - \frac{1}{x^2} = 0$
② $f_y = k^3x - \frac{1}{y^2} = 0$
$\overset{①/②}{\Longrightarrow} \frac{y}{x} = \frac{y^2}{x^2} \Rightarrow 1 = \frac{y}{x}$ or $\underline{y = x}$.

$y = x$ and ① $\Rightarrow k^3y - \frac{1}{y^2} = 0 \Rightarrow y^3 = k^{-3} \Rightarrow \underline{y = k^{-1} = x}$.

$\therefore \left(\frac{1}{k}, \frac{1}{k}\right)$ is a critical point.

$f_{xx} = 2x^{-3}$; $f_{xy} = k^3$; $f_{yy} = 2y^{-3}$ $\Rightarrow D(\frac{1}{k}, \frac{1}{k}) = \begin{vmatrix} 2k^3 & k^3 \\ k^3 & 2k^3 \end{vmatrix} = 3k^6 > 0$

for all $k \neq 0$. If $k > 0$, $f_{xx}(\frac{1}{k}, \frac{1}{k}) = 2k^3 > 0 \Rightarrow$ $\boxed{(\frac{1}{k}, \frac{1}{k}) \text{ is a minimum, } k>0}$

If $k < 0$, $f_{xx}(\frac{1}{k}, \frac{1}{k}) = 2k^3 < 0 \Rightarrow$ $\boxed{(\frac{1}{k}, \frac{1}{k}) \text{ is a maximum, } k<0}$

($k = 0$ is impossible since $-\frac{1}{x^2} \neq 0$.)

2. (7 points) Use Lagrange Multipliers to find the global extreme values of $f(x, y) = xy$ constrained by $4x^2 + y^2 = 16$. Draw a picture of the most important level curves to verify your final answer.

$C(x, y) = 4x^2 + y^2$

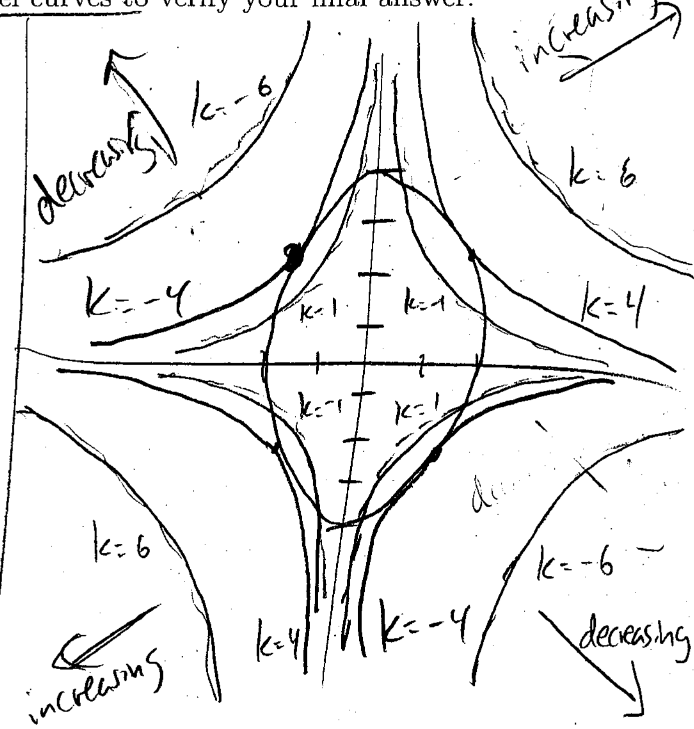
$\Rightarrow$ ① $y = 8\lambda x$
② $x = 2\lambda y$
③ $4x^2 + y^2 = 16$

$\frac{①}{②} \Rightarrow \frac{y}{x} = \frac{4x}{y} \Rightarrow y^2 = 4x^2$

③ $\Rightarrow 8x^2 = 16 \Rightarrow x = \pm\sqrt{2}$

$\Rightarrow y = \pm 2\sqrt{2}$.

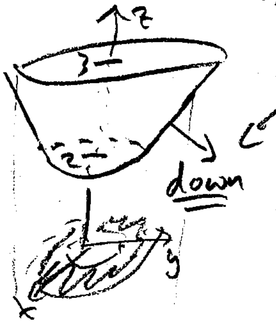
$\therefore$ $f(\sqrt{2}, 2\sqrt{2}) = 4$ is a global max.
$f(-\sqrt{2}, 2\sqrt{2}) = -4$ is a global min



$k = xy \Rightarrow y = \frac{k}{x}$ if $k \neq 0$.

$0 = xy \Rightarrow y = 0, x = 0$

3. (5 points) Find the flux of $\vec{F}(x,y,z) = \langle y, -x, z\rangle$ through the piece of $z^2 = x^2 + y^2$ that lies between z = 2 and z = 3, oriented away from the z - axis.



$$z = \sqrt{x^2+y^2} \Rightarrow d\vec{S} = \left\langle \frac{x}{r}, \frac{y}{r}, -1 \right\rangle dx\,dy$$

(down)

$R: 1 \le x^2+y^2 \le 9$

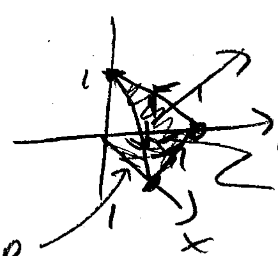
$$\Phi = \iint_R \langle y, -x, z\rangle \cdot \left\langle \frac{x}{r}, \frac{y}{r}, -1\right\rangle dx\,dy$$

$$= \iint_R -z\,dx\,dy$$

$$= \int_0^{2\pi}\int_2^3 -r^2\,dr\,d\theta = -2\pi \cdot \frac{r^3}{3}\Big|_2^3$$

$$= -2\pi\left(9 - \frac{8}{3}\right) = \boxed{\frac{-38\pi}{3}}$$

4. (5 points) Use Stokes Theorem to find the work done by $\vec{F}(x,y,z) = \langle z, y, z\rangle$ on a particle that moves around the triangle from vertex (1,0,0) to vertex (0,1,0) to vertex (0,0,1) and then back to (1,0,0).



$$\nabla \times \vec{F} = \langle 0, 1, 0\rangle$$

S is this piece of $x+y+z=1$ $\Rightarrow z = 1-x-y \Rightarrow d\vec{S} = \langle 1, 1, 1\rangle\,dx\,dy$

oriented up

$$\int_C \vec{F}\cdot d\vec{s} = \iint_S \langle 0,1,0\rangle \cdot d\vec{S} = \iint_R 1\,dx\,dy = \text{area}(R)$$

$$= \frac{1}{2}\cdot 1 \cdot 1 = \boxed{\frac{1}{2}}$$