Fall '15

Math 200 Quiz #1 25 points **Name:**________________________

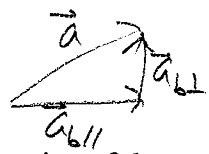
1) Let $\vec{a} = \langle 1,2,2 \rangle$ and $\vec{b} = 3\hat{\mathbf{i}} - 2\hat{\mathbf{j}} + \hat{\mathbf{k}}$ for all parts of this problem. Show work, but you need not repeat work.

1A) Find the work done by the constant force field $\vec{a}$ on a particle with displacement $\vec{b}$. (2 points)

$$W = \vec{a} \cdot \vec{b} = \langle 1,2,2 \rangle \cdot \langle 3,-2,1 \rangle = 3 \cdot 1 + 2(-2) + 2 \cdot 1 = 1 \text{ work unit}$$

1B) Find $\vec{a}_{\vec{b}\parallel}$, the projection of $\vec{a}$ onto $\vec{b}$, **and** $\vec{a}_{\vec{b}\perp}$, the normal component of $\vec{a}$ to $\vec{b}$. (4 points)

$$\vec{a}_{\vec{b}\parallel} = \frac{\vec{a}\cdot\vec{b}}{\vec{b}\cdot\vec{b}}\vec{b} = \frac{\langle 3,-2,1 \rangle}{9+4+1} = \boxed{\frac{\langle 3,-2,1 \rangle}{14}}$$



$$\vec{a}_{\vec{b}\perp} + \vec{a}_{\vec{b}\parallel} = \vec{a}$$

$$\Rightarrow \vec{a}_{\vec{b}\perp} = \vec{a} - \vec{a}_{\vec{b}\parallel} = \frac{\langle 14,28,28 \rangle - \langle 3,-2,1 \rangle}{14}$$

$$\Rightarrow \boxed{\vec{a}_{\vec{b}\perp} = \frac{\langle 11,30,27 \rangle}{14}}$$

1C) Find the cosine of the angle between $\vec{a}$ and $\vec{b}$. (2 points)

$$\cos\theta = \frac{\vec{a}\cdot\vec{b}}{|\vec{a}|\,|\vec{b}|} = \boxed{\frac{1}{3\sqrt{14}}}$$

$$|\vec{a}| = \sqrt{\vec{a}\cdot\vec{a}} = \sqrt{1+4+4} = 3$$

1D) Find the equation, in standard form, of the plane that is parallel to $\vec{a}$ and $\vec{b}$, and that passes through the point (1, 1, 1). (4 points)

$$\begin{array}{r} \vec{a} = \langle 1,2,2 \rangle \\ \times\ \vec{b} = \langle 3,-2,1 \rangle \\ \hline \langle 6,5,-8 \rangle \end{array}$$

$$\langle 6,5,-8 \rangle \cdot \langle 1,1,1 \rangle = 3.$$

$\therefore$ the equation of the plane is $\boxed{6x + 5y - 8z = 3}$

1E) Find the flux of the constant vector field $\vec{v} = \langle 1,-2,1 \rangle$ through ~~the plane from #1D~~ the //ogram spanned by $\vec{a}$ and $\vec{b}$, oriented from $\vec{a}$ to $\vec{b}$. (~~2~~ points)

$$\Phi = \vec{v} \cdot (\vec{a}\times\vec{b}) = \langle 1,-2,1 \rangle \cdot \langle 6,5,-8 \rangle = 6 - 10 - 8 = \boxed{-12}$$

1F) Find the distance from the origin to the plane from #1D. (~~1~~ 2 points)

$$\frac{6x + 5y - 8z = 3}{\sqrt{36+25+64}} \quad \frac{3}{\sqrt{125}}$$

$\Rightarrow$ Distance is $\boxed{\frac{3}{\sqrt{125}}}$ units

$\Leftarrow$ the component of any planar point $\langle x,y,z \rangle$ on the normal

$$\frac{\langle 6,5,-8 \rangle \cdot \langle x,y,z \rangle}{|\langle 6,5,-8 \rangle|}$$

2) Find a parametrization for the line that contains the points $P = \left(1, \frac{\pi}{2}, 3\right)_C$ and $Q = \left\langle 2\sqrt{2}, \frac{\pi}{4}, \frac{\pi}{2} \right\rangle_S$. Express your answer in two ways: as a vector parameterization and as parametric equations. Hint: Convert first. (~~6~~ points) 8

$P = (1, \frac{\pi}{2}, 3)_C$: $x = 1 \cdot \cos(\pi/2) = 0$

$y = 1 \cdot \sin(\pi/2) = 1 \Rightarrow P: (0,1,3)$

$z = 3$

$Q = (2\sqrt{2}, \frac{\pi}{4}, \frac{\pi}{2})_S$: $x = 2\sqrt{2} \sin\frac{\pi}{2} \cos\frac{\pi}{4} = 2\sqrt{2} \cdot \frac{\sqrt{2}}{2} = 2$

$y = 2\sqrt{2} \sin(\pi/2) \sin\pi/4 = 2\sqrt{2} \cdot \frac{\sqrt{2}}{2} = 2 \Rightarrow Q: (2,2,0)$

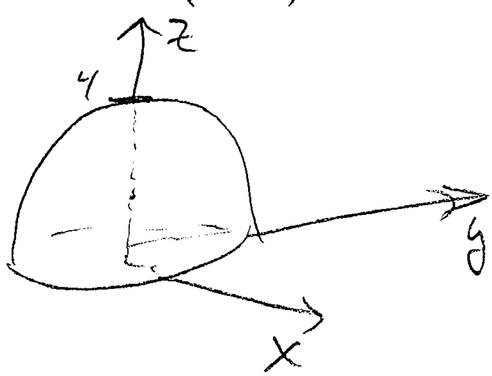
$z = 2\sqrt{2} \cos(\frac{\pi}{2}) = 0$

$\therefore \vec{r}(t) = \vec{P} + t(\vec{PQ}) = \langle 0,1,3 \rangle + t(\langle 2,2,0 \rangle - \langle 0,1,3 \rangle)$

$\Rightarrow \boxed{\vec{r}(t) = \langle 2t, 1+t, 3-3t \rangle, t \in \mathbb{R}}$ is a vector parameterization

and $\boxed{x = 2t, \ y = 1+t, \ z = 3-3t, \ t \in \mathbb{R}}$ are parametric equations.

3) Sketch the surface $z = 4 - (x^2 + y^2)$ with coordinate axes labeled using positive orientation. (~~3~~ points) 2



4) Write $x^2 + y^2 + (z-2)^2 = 4$ using spherical coordinates and simplify. (~~3~~ points) 2

$x^2 + y^2 + z^2 - 4z + 4 = 4 \Rightarrow \rho^2 = 4\rho\cos\phi \Rightarrow \boxed{\rho = 4\cos\phi}$