Math 200 Quiz #2 25 points Name:______________________

1) $\vec{r}(t) = \langle e^t, e^{-t}, \sqrt{2}t \rangle$ for all parts of this problem. You need not repeat work.

1A) Find the arc length of the trace of $\vec{r}(t)$ from t = 0 to t = 4. (~~4~~ 5 points)

$$\|\vec{r}\,'(t)\|^2 = e^{2t} + e^{-2t} + 2 = (e^t + e^{-t})^2$$

$$\Rightarrow s = \int_0^4 e^t + e^{-t}\,dt = e^t - e^{-t}\Big|_0^4 = \boxed{e^4 - e^{-4}} \text{ or } \boxed{2\sinh(4)}$$

[Note: $\vec{r}\,'(t) = \langle e^t, -e^{-t}, \sqrt{2} \rangle$ and $\vec{r}\,''(t) = \langle e^t, e^{-t}, 0 \rangle$]

1B) Find the equation of the osculating plane for the trace of $\vec{r}(t)$ at t = 0. (~~3~~ 4 points)

$$\vec{r}\,'(0) = \langle 1, -1, \sqrt{2} \rangle$$

$$\times\ \vec{r}\,''(0) = \langle 1, 1, 0 \rangle$$

$$\vec{n} = \langle -\sqrt{2}, \sqrt{2}, 2 \rangle$$

$$\vec{n} \cdot \vec{r}(0) = \langle -\sqrt{2}, \sqrt{2}, 2 \rangle \cdot \langle 1, 1, 0 \rangle = 0$$

$\vec{r}(0) = \langle 1, 1, 0 \rangle$

$\Rightarrow$ the equation of the osculating plane is

$$\boxed{-\sqrt{2}x + \sqrt{2}y + 2z = 0} \text{ or } \boxed{x - y - \sqrt{2}z = 0}$$

1C) Find the curvature for the trace of $\vec{r}(t)$ at t = 0. (~~3~~ 4 points)

$$\kappa(0) = \frac{|\vec{v} \times \vec{a}|}{|\vec{v}|^3} = \frac{|\langle -\sqrt{2}, \sqrt{2}, 2 \rangle|}{\sqrt{1+1+2}^{\,3}} = \frac{\sqrt{2+2+4}}{8} = \boxed{\frac{\sqrt{2}}{4}}$$

1D) If the trace of $\vec{r}(t)$ represents the path of a particle, is that particle speeding up at t = 0? Defend. (2 points)

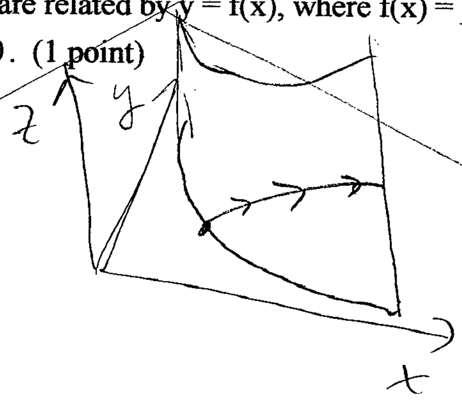
$\vec{v}(0) \cdot \vec{a}(0) = 0$, so the particle is not speeding up because $\vec{a}(0) \perp \vec{v}(0)$. (The particle is not slowing either)

1E) The x and y coordinates of the trace of $\vec{r}(t)$ are related by y = f(x), where f(x) = $\underline{\ \frac{1}{x}\ }$. Use this relation to help sketch the trace ~~of trace of~~ $\vec{r}(t)$ for $t \geq 0$. (1 point)

$$y = \frac{1}{x}$$

$$e^{\frac{z}{\sqrt{2}}} = x$$

$$\Rightarrow z = \sqrt{2}\ln x$$



2) Find and simplify $\int_{-\pi}^{\pi} \langle t^3, \sin^2(t), \cos(4t)\cos(t) \rangle dt$. Show work to defend your answer. (6 points)

$$= \left\langle \underbrace{\int_{-\pi}^{\pi} t^3\, dt}_{\text{ODD}},\ \int_{-\pi}^{\pi} \frac{1-\cancel{\cos(2t)}}{2}\, dt,\ \int_{-\pi}^{\pi} \frac{\cancel{\cos(5t)} + \cancel{\cos(3t)}}{2}\, dt \right\rangle$$

2T →0, 5T →0, 3T →0

$$= \left\langle 0,\ \int_{-\pi}^{\pi} \tfrac{1}{2}\, dt,\ 0 \right\rangle$$

$$= \boxed{\langle 0, \pi, 0 \rangle}$$

3) Find $\cancel{\alpha}'(0)$ $f'(0)$ if $\cancel{\alpha(t)}$ $f(t)$ $= \vec{p}(t) \cdot \langle \sin(3t), \cos(3t), t^2 \rangle$, $\vec{p}(0) = \langle 1,1,1 \rangle$, and $\vec{p}\,'(0) = \langle 2,3,4 \rangle$. Show work to defend your answer. (6 points)

$$f'(0) = \vec{p}\,'(0) \cdot \langle 0, 1, 0 \rangle + \vec{p}(0) \cdot \langle 3\cos(0), -3\sin(0), 2 \cdot 0 \rangle$$

$$= 3 \qquad + \ 3$$

$$= 6$$

#3

+ Max of 2 for any deducted points.