Math 200 Quiz #3 25 points Name:

1A) Find the gradient of $f(x,y,z)=\cosh(xyz)$, that is, find ∇f. (3 points)

$$\left.\begin{aligned}\frac{\partial f}{\partial x} &= yz\sinh(xyz)\\ \frac{\partial f}{\partial y} &= xz\sinh(xyz)\\ \frac{\partial f}{\partial z} &= xy\sinh(xyz)\end{aligned}\right\} \Longrightarrow \boxed{\nabla f = \sinh(xyz)\langle yz, xz, xy\rangle}$$

1B) Find and simplify $f_{xx}(x,y,z)$ and $\dfrac{\partial^2 f}{\partial y \partial x}$. How many other ***different*** second partial derivatives of f(x, y, z) are there? 4. (5 points)

$$\boxed{\begin{aligned} f_{xx} &= (yz)^2\sinh(xyz)\\ \frac{\partial^2 f}{\partial y\partial x} = f_{xy} &= z\sinh(xyz) + xyz^2\cosh(xyz)\end{aligned}}$$

Four different 2^{nd} partials: $f_{yy}, f_{zz}, f_{xz}, f_{yz}$

Note: $f_{xz}=f_{zx}$, $f_{yz}=f_{zy}$, and $f_{xy}=f_{yx}$

2) Find the limit if possible, or prove the limit does not exist. Show organized work to defend. (5 points)

2A) $\displaystyle\lim_{(x,y)\to(0,0)} \frac{1-e^{x^2+y^2}}{x^2+y^2}$ $= \displaystyle\lim_{\substack{r\to 0^+\\ \theta \text{ free}}} \frac{1-e^{r^2}}{r^2} \underset{(L'H)}{=} \lim_{r\to 0^+} \frac{-\cancel{2r}e^{r^2}}{\cancel{2r}} = \boxed{-1}$

2B) $\displaystyle\lim_{(x,y)\to(1,4)} \frac{(x-1)^2+2(y-4)^2}{2(x-1)^2+(y-4)^2}$

$$\lim_{(1,y)\to(1,4^+)} \frac{0+2\cancel{(y-4)^2}}{0+\cancel{(y-4)^2}} = 2$$

$$\lim_{(x,4)\to(1^+,4)} \frac{\cancel{(x-1)^2}+2(0)}{2\cancel{(x-1)^2}+0} = \frac{1}{2}$$

$2 \neq \frac{1}{2} \Rightarrow$ the limit DNE.

3) Set up an integral that represents the mass of the wire that lies on $x^2 + y^2 = 4$ from $(0,-2)$ **clockwise** to $(-2,0)$ in the xy - plane if its linear density is $\delta(x,y) = y^2$, and then **evaluate** the integral. (6 points)

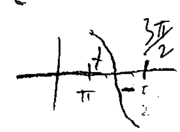
$\text{Mass} = \int_C y^2\, ds.$

$\vec{r}(t) = 2\langle \sin(t), \cos(t)\rangle, \quad \pi \le t \le \frac{3\pi}{2}$

$\Rightarrow \vec{r}\,'(t) = 2\langle \cos(t), -\sin t\rangle.$

$\Rightarrow ds = \sqrt{4\cos^2 t + 4\sin^2 t}\, dt = 2\, dt.$

$\therefore \text{Mass} = \int_{\pi}^{\frac{3\pi}{2}} (4\cos^2 t)\, 2\, dt = 4\int_{\pi}^{\frac{3\pi}{2}} 1 + \overbrace{\cancel{\cos(2t)}}^{\to 0}\, dt$

$\frac{1}{2}T:$ 

$\Rightarrow \text{Mass} = 4 \cdot \frac{\pi}{2} = 2\pi$ units of mass.

4/ 3) Evaluate $I = \int_C z\,dx + \cos(x)\,dy + \sin(y)\,dz$ if C is the curve from ~~(0, 0, 0)~~ $(-2,-8,-2)$ to (2, 8, 2) on the trace of $\vec{r}(t) = \langle t, t^3, t\rangle$.

$\sqrt[3]{y}$

(6 points)

$-2 \le t \le 2$

$\vec{r}\,'(t) = \langle 1, 3t^2, 1\rangle$

$I = \int_{-2}^{2} \underbrace{\cancel{t}}_{\text{ODD}}^{\to 0} + \underbrace{\cos(t)\cdot 3t^2}_{\substack{\text{even}\cdot\text{even} \\ \text{even}}} + \underbrace{\cancel{\sin(t)}}_{\text{ODD}}^{\to 0}\, dt$

$\frac{d}{dt}$		$\int dt$
$3t^2$	+	$\cos(t)$
$6t$	−	$\sin(t)$
6	+	$-\cos(t)$
0		$-\sin(t)$

$= 2\left(3t^2\sin(t) + 6t\cos(t) - 6\sin(t)\right)\Big|_0^2$ ← makes all terms 0,

$= 2\left(6\sin(2) + 12\cos(2)\right)$

$= \boxed{12\sin(2) + 24\cos(2)}$