Math 200 Quiz #4 25 points Name:______________________________

1) Find the tangent plane, in ax + by + cz = d form, for the graph of $z = f(x,y) = x^2y - \cos(xy)$ at $(1,0,-1)$ two different ways. (8 points)

1A) Do not use ~~a~~ the gradient of a new function.

$$f_x = 2xy + y\sin(x,y)$$
$$f_y = x^2 + x\sin(xy)$$
$$\Rightarrow \nabla f(1,0) = \langle 0,1\rangle;$$

$$\therefore\ z+1 = 0(x-1) + 1(y-0) \Rightarrow \boxed{y - z = 1} \text{ is the T.P.}$$

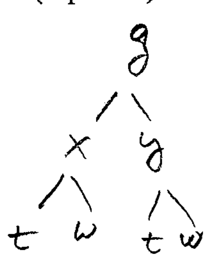
1B) Use ~~a~~ the gradient of a new function. Write the new function.

Let $F(x,y,z) = z - x^2y + \cos(xy)$. Then the graph of $z = f(x,y)$ is the level surface $F = 0$. $\nabla F = \langle -2xy - y\sin(xy), -x^2 - x\sin(xy), 1\rangle$

$\Rightarrow \nabla F(1,0,-1) = \langle 0-0, -1-0, 1\rangle$ is $\perp$ to the T.P. @ $(1,0,-1)$.

$\langle 0,-1,1\rangle \cdot \langle 1,0,-1\rangle = -1$, so $\boxed{-y + z = -1}$ is the T.P.

2) Find $\dfrac{\partial g}{\partial t}$ at t = 2 and w = 3 if $g(x,y) = \cos(xy)$, $x = x(t,w)$, with $x(2,3) = 1$, $x_t(2,3) = 2$, and $y = \dfrac{\pi t w}{4}$. (5 points)



$x(2,3) = 1$

$y(2,3) = \frac{3\pi}{2}$

$$\frac{\partial g}{\partial t} = g_x\left(1,\tfrac{3\pi}{2}\right)\cdot x_t(2,3) + g_y\left(1,\tfrac{3\pi}{2}\right)\cdot y_t(2,3)$$
$$= -y\sin(xy)\Big|_{(1,\frac{3\pi}{2})} \cdot 2 + \left(-x\sin(xy)\right)\Big|_{(1,\frac{3\pi}{2})} \cdot \frac{w\pi}{4}\Big|_{w=3}$$
$$= -\frac{3\pi}{2}(-1)\cdot 2 + \left(-1(-1)\right)\cdot\frac{3\pi}{4}$$
$$= 3\pi + \frac{3\pi}{4} = \boxed{\frac{15\pi}{4}}$$

$\frac{dm}{m}\cdot 100 = 10\%$ $\qquad\qquad$ $\frac{dv}{v}\cdot 100 = 4\%$

3) Suppose $f(m,v)=\frac{1}{2}mv^2$ where m is measured with a 10% maximum percent error and v is measured with a 4% maximum percentage error. Use differentials to estimate the maximum percentage error of f(m, v) when ~~m~~ = ~~10~~ 4 and v = ~~2~~ 100. (4 points)

$$\text{Max \% error} = \frac{df}{f}\cdot 100 = \left(\frac{\frac{1}{2}v^2\,dm + mv\,dv}{\frac{1}{2}mv^2}\right)\cdot 100 = \left(\frac{dm}{m}\cdot 100\right) + 2\left(\frac{dv}{v}\cdot 100\right)$$

$$= 10\% + 8\% = \boxed{18\% \text{ max \% error}}$$

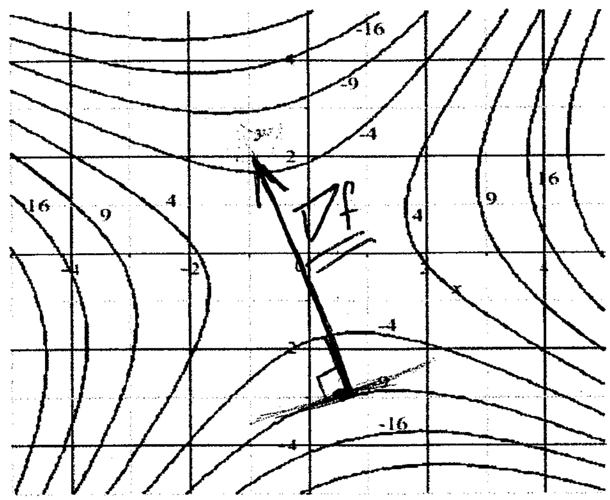
(No matter what m or v are.)

3) The contour map on the right is for z = f(x, y). Show work as you estimate the magnitude of $\nabla f(0.5,-3)$, and then sketch $\nabla f(0.5,-3)$ on the contour plot and (4 points)

$$|\nabla f(0.5,-3)| \approx \frac{-4-(-9)}{1} = 5$$

(Note: $\frac{-9-(-16)}{1} = 7$ or

$\frac{-4-(-16)}{2} = 6$ are reasonable estimates as well)



4) Find the directional derivative of $g(x,y)=\sin(xy^2)$ for the input $P=(\pi,1)$ in the direction $\langle 4,3\rangle$.
(4 points)

$$\nabla g(\pi,1) = \langle y^2\cos(xy^2),\ 2xy\cos(xy^2)\rangle\Big|_{(\pi,1)} = \langle -1, -2\pi\rangle.$$

$$\hat{u} = \frac{\langle 4,3\rangle}{5},\ \text{so}\ D_{\hat{u}}\,g(\pi,1) = \langle -1,-2\pi\rangle\cdot\frac{\langle 4,3\rangle}{5}$$

$$= \boxed{\frac{-4-6\pi}{5}}$$