Math 200 Quiz #5 25 points Name:

1) Evaluate the following line integrals. Use the FTCLI whenever possible. If impossible, show how you know before using test #1 methods to find the line integral. (8 points)

1A) $\int_C \vec{F}\cdot d\vec{s}$ if $\vec{F}=\langle \underset{M}{e^x y}, \underset{N}{e^x+2y}\rangle$ and C is the curve $\vec{r}(t)=\langle 1+\cos t, 1+\sin t\rangle$ for $0\le t\le \pi$. (7 points)

$\frac{\partial N}{\partial x}=e^x$ $\Rightarrow$ potential exists. $\phi(a,b)=\int_0^a e^x\cdot 0\,dx+\int_0^b e^a+2y\,dy = 0+\left(ye^a+y^2\right)\Big|_0^b$
$\frac{\partial M}{\partial y}=e^x$ $\qquad = be^a+b^2.$

$\Longrightarrow$ $\underline{\phi(x,y)=ye^x+y^2}$ $\qquad$ Check: $\nabla\phi=\langle ye^x, e^x+2y\rangle$ ✓

$\therefore$ $\underline{\text{FTCLI}}$ $\Rightarrow$ $\int_C \vec{F}\cdot d\vec{s} = \phi(\vec{r}(\pi))-\phi(\vec{r}(0))$

$= \phi(0,1)-\phi(2,1)$

$=(1+1)-(e^2+1) = \boxed{1-e^2}$

1B) $I=\int_C (\underset{M}{2x-3y})dx+\underset{N}{x}dy$ ~~if~~ and C is the curve ~~that moves~~ oriented from $(-2,4)$ to $(2,4)$ on the path $y=x^2$. (6 points)

$\frac{\partial N}{\partial x}=1$ $\Rightarrow$ FTCLI does $\underline{\text{not}}$ apply b/c the curl test fails.
$\ne$
$\frac{\partial M}{\partial y}=-3$

$\vec{r}(t)=\langle \overset{x}{t},\overset{y}{t^2}\rangle$ $\Rightarrow \vec{r}\,'(t)=\langle 1,2t\rangle$, so $dx=1\,dt$ and $dy=2t\,dt$.
$-2\le t\le 2$

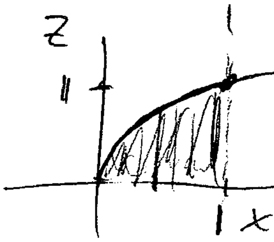
$I=\int_{-2}^{2}(2t-3t^2)+t\cdot 2t\,dt = \int_{-2}^{2}\underset{\text{ODD}}{\cancel{2t}}^{\,0}-\underset{\text{Even}}{t^2}\,dt = -2\int_0^2 t^2\,dt.$

$\therefore$ $I=-2\,\frac{t^3}{3}\Big|_0^2 = \boxed{\frac{-16}{3}}$

2) Evaluate $I = \int_0^1 \int_{z^2}^1 \int_{-z}^{z} \cancel{\sin(y)}^{\,0} + \sin(x^2)\, dydxdz$. (6 points)

ODD Even

$$I = \int_0^1 \int_{z^2}^1 2y \sin(x^2) \Big|_0^z dx\,dz = \int_0^1 \int_{z^2}^1 2z \sin(x^2)\, dx\, dz$$



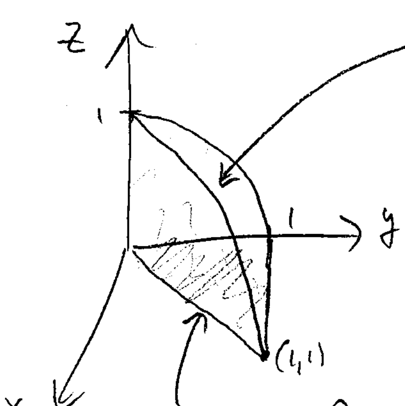
$x = z^2 \Rightarrow z = \sqrt{x}$

(switch order) $= \int_0^1 \int_0^{\sqrt{x}} 2z \sin(x^2)\, dz\, dx$

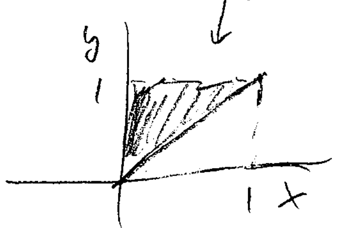
$$= \int_0^1 z^2 \sin(x^2) \Big|_0^{\sqrt{x}} dx$$

$$= \int_0^1 x \sin(x^2)\, dx = \frac{-\cos(x^2)}{2}\Big|_0^1 = \boxed{\frac{1-\cos(1)}{2}}$$

3) **Use a double integral** to find the volume of the solid in the first octant (x,y, and z are all nonnegative) bounded by $z = 0$, $x = 0$, $y = x$, and by $z = 1 - \sqrt{y}$. (6 points)



R = region of integration.



$$\text{Vol} = \int_0^1 \int_x^1 1 - \sqrt{y}\; dy\, dx$$

$$= \int_0^1 y - \frac{2}{3} y^{\frac{3}{2}} \Big|_x^1 dx$$

$$= \int_0^1 \frac{1}{3} - x + \frac{2}{3} x^{3/2}\, dx$$

$$= \frac{1}{3} - \frac{x^2}{2} + \frac{4}{15} x^{\frac{5}{2}} \Big|_0^1$$

$$= \frac{1}{3} - \frac{1}{2} + \frac{4}{15} = \frac{10 - 15 + 8}{30} = \boxed{\frac{1}{10}}$$ Cubic unit