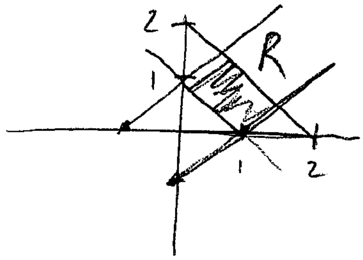
Math 200 Quiz #7 25 points Name:________________

1) Use substitution to calculate $I = \iint_R \frac{(x-y)^2}{\sqrt{x+y}}\,dxdy$ if R is the region in the xy - plane bounded by $x+y=1$, $x+y=2$, $x-y=1$, and $x-y=-1$. (7 points)



$u = x-y$; $-1 \le u \le 1$

$v = x+y$; $1 \le v \le 2$

$\Rightarrow \frac{\partial(u,v)}{\partial(x,y)} = \begin{vmatrix} 1 & -1 \\ 1 & 1 \end{vmatrix} = 2$

and $dxdy = \frac{\partial(x,y)}{\partial(u,v)}\,dudv$

$\therefore\ I = \int_1^2 \int_{-1}^1 \frac{u^2}{\sqrt{v}} \cdot \frac{1}{2}\,du\,dv$

(even in u)

$I = \int_1^2 \frac{u^3}{3\sqrt{v}}\Big|_0^1 dv$

$= \frac{1}{3}\int_1^2 v^{-1/2}\,dv$

$= \frac{1}{3}\cdot 2\, v^{1/2}\Big|_1^2$

$= \boxed{\frac{2}{3}\left(\sqrt{2}-1\right)}$

2) Find the mass of the surface S parameterized by $\vec{\alpha}(u,p) = \langle \cos u, \sin u, \sin p\rangle$ for $0 \le u \le \frac{\pi}{2}$, and $0 \le p \le \frac{\pi}{2}$ if the density is $\delta(x,y,z) = x^2+y^2+z^2$ grams per square meter. (6 points)

$\delta(u,p) = 1 + \sin^2 p$

$\vec{\alpha}_u = \langle -\sin u, \cos u, 0\rangle$

$\times\ \vec{\alpha}_p = \langle 0, 0, \cos p\rangle$

$\langle \cos u \cos p, \sin u \cos p, 0\rangle$

$\Rightarrow dS = \sqrt{\cos^2 u \cos^2 p + \sin^2 u \cos^2 p}\,du\,dp$

$= |\cos p|\,du\,dp.$

$= \cos p\,du\,dp$ for $0 \le p \le \pi/2$

$\text{Mass} = \int_0^{\pi/2}\int_0^{\pi/2} (1+\sin^2 p)\cos p\,du\,dp$

$w = \sin p$

$= \int_0^{\pi/2} 1\,du \cdot \int_0^1 1 + w^2\,dw$

$= \frac{\pi}{2}\cdot\left(w + \frac{w^3}{3}\right)\Big|_0^1 = \frac{\pi}{2}\cdot\frac{4}{3}$

$= \boxed{\frac{2\pi}{3}}$ grams.

3) Find the flux of $\vec{F}(x,y,z)=\langle 2x+y, x-2y, 2\rangle$ through the graph of $f(x,y)=2x^2-2xy-2y^2$, oriented up, if $1\le x^2+y^2\le 4$. (6 points)

special case: $d\vec{S}=\pm\langle -f_x,-f_y,1\rangle\,dx\,dy$

$$d\vec{S}=\langle -4x+2y,\ 2x+4y,\ 1\rangle\,dx\,dy$$

$$\Phi=\iint_S \vec{F}\cdot d\vec{S}\ ;\quad \vec{F}\cdot d\vec{S}=(-8x^2+2y^2+2x^2-8y^2+2)\,dx\,dy$$

$$=\iint_{1\le x^2+y^2\le 4} -6x^2-6y^2+2\,dx\,dy$$

$$\underset{\text{(polar)}}{=}\int_0^{2\pi}\int_1^2 -6r^3\,dr\,d\theta+2(\text{Area }1\le x^2+y^2\le 4)$$

$$=2\pi\cdot\left(\frac{-6r^4}{4}\right)\Bigg|_1^2+2\left[4\pi-\pi\right]=-3\pi(16-1)+2(3\pi)$$

$$=-39\pi$$

4) Find the flux of $\vec{F}(x,y,z)=\langle x,z,y\rangle$ through the piece of the cylinder $x^2+y^2=4$ that lies in the first octant below $z=5$. This surface is oriented away from the z – axis. (6 points)

$r=2$ $\quad 0\le\theta\le\frac{\pi}{2}$ $\quad 0\le z\le 5$

special case:

$d\vec{S}=\pm\langle\cos\theta,\sin\theta,0\rangle\cdot 2\,d\theta\,dz$

$\vec{F}=\langle 2\cos\theta, z, 2\sin\theta\rangle$

$\Rightarrow \vec{F}\cdot d\vec{S}=(4\cos^2\theta+2z\sin\theta)\,d\theta\,dz$

$$\Phi=\int_0^{\pi/2}\int_0^5 (4\cos^2\theta+2z\sin\theta)\,dz\,d\theta$$

$$=\int_0^{\pi/2} 10+10\cancel{\cos(2\theta)}^{\,0}+\left(z^2\Big|_0^5\right)\sin\theta\,d\theta$$

$$=5\pi+25\left(-\cos\theta\Big|_0^{\pi/2}\right)$$

$$=\boxed{5\pi+25}$$

Note: If flux of $\vec{F}$ was through $x^2+y^2+z^2=4$, then

$d\vec{S}=\frac{\langle x,y,z\rangle}{2}\cdot 4\sin\phi\,d\phi\,d\theta$

$$\Rightarrow\Phi=\int_0^{\pi/2}\int_0^{\pi/2} 2(x^2+2yz)\sin\phi\,d\phi\,d\theta$$

$$=\int_0^{\pi/2}\int_0^{\pi/2} 2(4\sin^2\phi\cos^2\theta+8\sin\phi\sin\theta\cos\phi)\sin\phi\,d\phi\,d\theta$$

$=$ etc.