Math 200 Quiz #8 25 points Name:

1) Find the curl and divergence of $\vec{F} = \langle \cancel{xz, zy}, xy \rangle$ $\langle x^2y, \frac{x^3}{3}, xy \rangle$. Label each answer with appropriate notation. (4 points)

$$\nabla \cdot \vec{F} = \frac{\partial x^2y}{\partial x} + \frac{\partial \frac{x^3}{3}}{\partial y} + \frac{\partial (xy)}{\partial z} = 2xy + 0 + 0 = \boxed{2xy} \text{ (Divergence)}$$

$$\begin{array}{ccc} \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x^2y & \frac{x^3}{3} & xy \end{array}$$

$$\langle x-0, 0-y, x^2-x^2 \rangle$$

$$\Rightarrow \boxed{\nabla \times \vec{F} = \langle x, -y, 0 \rangle} \text{ (Curl)}$$

2) Use Stokes' Theorem **once** to find $I = \oint_{Bd(S)} x^2y\,dx + \frac{x^3}{3}dy + xy\,dz$ if S is the intersection of $z = y^2 - x^2$ and $x^2 + y^2 \le 1$, and S is oriented up. (4 points)

R +

special graph case

$d\vec{S} = \pm \langle 2x, -2y, 1 \rangle dx\,dy$

$$I \underset{S.T.}{=} \iint_S \left(\nabla \times \langle x^2y, \frac{x^3}{3}, xy \rangle \right) \cdot d\vec{S}$$

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$$= \iint_{x^2+y^2 \le 1} \langle x, -y, 0 \rangle \cdot \langle 2x, -2y, 1 \rangle \, dx\,dy \quad ; \quad 2x^2 + 2y^2 = 2r^2$$

$$= \int_0^{2\pi} \int_0^1 2r^3 \, dr\,d\theta = 4\pi \cdot \frac{r^4}{4} \Big|_0^1 = \pi$$

3) Use Stokes' Theorem **once** to find $I = \iint_S (\nabla \times \vec{F}) \cdot d\vec{S}$ if $\vec{F} = \langle e^z - y, e^z + x, \cos(xz) \rangle$ and S is the upper hemisphere, $\rho = 1$ with $z \ge 0$ oriented **down**. (5 points)

$$I \underset{S.T.}{=} -\oint_{\substack{x^2+y^2=1 \\ \circlearrowleft}} \vec{F} \cdot d\vec{s} \quad ; \quad C: \vec{r}(t) = \langle \cos t, \sin t, 0 \rangle, \ 0 \le t \le 2\pi$$

$$\vec{r}'(t) = \langle -\sin t, \cos t, 0 \rangle$$

$$I = -\int_0^{2\pi} \langle e^0 - \sin t, e^0 + \cos t, \cos(0) \rangle \cdot \langle -\sin t, \cos t, 0 \rangle \, dt$$

$$= -\int_0^{2\pi} \overset{\to 0;\ \text{IT}}{\cancel{-\sin t}} + \sin^2 t + \overset{\to 0;\ \text{IT}}{\cancel{\cos t}} + \cos^2 t \, dt = -\int_0^{2\pi} 1 \, dt$$

$$= -2\pi$$

4) Use the Divergence Theorem **once** to find I= $\oiint_{Bd(T)} \vec{F}\cdot d\vec{S}$ if $\vec{F} = \langle 2xy, 3y^2, -2zy\rangle$ and T is the solid cube in the first octant with four of its vertices at (0, 0, 0), (2, 0, 0), (0, 2, 0), and (0, 0, 2). (6 points)

$$I \underset{\text{D.T.}}{=} \iiint_T \nabla\cdot\vec{F}\, dV = \int_0^2\int_0^2\int_0^2 2y+6y-2y\, dx\, dz\, dy$$

$$\Rightarrow I = \int_0^2 6y\cdot 4\, dy = 12y^2\Big|_0^2 = \boxed{48}$$

5) Use the divergence theorem to find I = $\iint_S 3x^2+xy+yz\, dS$ if S is $x^2+y^2+z^2=4$. (6 points)

Special spherical case

$$I = \iint_S \langle 3x, x, y\rangle\cdot\langle x,y,z\rangle\, dS$$

Multiply and divide by 2 to create $d\vec{S} = \frac{\langle x,y,z\rangle}{2} dS$

$$I = 2\iint_S \langle 3x, x, y\rangle\cdot d\vec{S}$$

$$\Rightarrow I \underset{\text{D.T.}}{=} 2\iiint_{\rho\le 2} 3\, dV = 6(\text{vol } \rho\le 2) = 6\cdot\frac{4}{3}\pi\cdot 2^3 = 64\pi$$