Test #1A 100 points Math 200 Name:_______________________________________

Simplify your final answers. Show organized work. Defend all answers.

1) Find the mass of a wire with density $\delta = \dfrac{xy}{\sqrt{1+\cos^2(x)}}$ grams per cm if it is located on the path $y = \sin(x)$ from the point $(0,0)$ to the point $(\pi,0)$. (10 points)

2) Find and evaluate an integral representing the work done by $\vec{F} = \langle -y, x, z+x \rangle$ on a particle that moves on the curve parameterized by $\vec{r}(t) = \langle \sin t, \sin t, \cos t \rangle$ from $t=0$ to $t=\pi$. (10 points)

3) Evaluate I = $\int_C 4xy\,dx + 2y\,dy$ if C is a line segment from (1, 3) to (1, 1) and a curve from (1, 1) to (3, 9) on $y = x^2$. (10 points)

4) Let $\vec{f}(t)=\langle P(t),Q(t)\rangle$ and $\vec{g}(t)=\langle R(t),W(t)\rangle$. Prove that $\left(\vec{f}\cdot\vec{g}\right)'=\vec{f}'\cdot\vec{g}+\vec{f}\cdot\vec{g}'$. (5 points)

5A) Find all the second partial derivatives of $f(x,y)=x^y$. Hint: $x^y=e^{y\ln(x)}$. (6 points)

5B) What is $\nabla f\left(2,2\right)$ if $f(x,y)=x^y$? (4 points)

6) Find the equation of the plane that contains the points P = $(1,1,0)$, Q = $(1,0,1)$, and R = $(0,1,2)$. Write your final answer in standard form. (10 points)

7) Let $\vec{a} = \langle 1, 2, 2 \rangle$, $\vec{b} = 2\hat{i} - \hat{j} + 3\hat{k}$, and $\vec{c} = \langle = -2, 1, -1 \rangle$. (25 points; #7A is 4 points, the others are 3 points)

7A) Find the area of the parallelogram spanned by $\vec{b}$ and $\vec{c}$.

7B) Find the work done on an object with displacement $\vec{a}$ by force $\vec{b}$.

7C) Find the flux of $\vec{a}$ through the parallelogram spanned by $\vec{b}$ and $\vec{c}$.

7D) Find the volume of the box spanned by $\vec{a}$, $\vec{b}$, and $\vec{c}$.

7E) Find the component of $\vec{a}$ onto $\vec{b}$.

7F) Find the projection of $\vec{a}$ onto $\vec{b}$.

7G) Find an equation for the osculating plane at the point $r(0) = (1, 0, -1)$ if $\vec{r}'(0) = \vec{b}$ and $\vec{r}''(0) = \vec{c}$.

7H) Find the curvature at $r(0) = (1, 0, -1)$ if $\vec{r}'(0) = \vec{b}$ and $\vec{r}''(0) = \vec{c}$.

8) Find the arc length of $\vec{r}(t) = \langle \cos t, \cosh t, \sin t \rangle$ from t = 0 to t = ln(3). (10 points)

9) Sketch the surface corresponding to $y = -\sqrt{z^2 + x^2}$. Label the axes. (5 points)

10) Use the contour plot of f(x, y) to estimate $f_y(5,5)$.
Draw an appropriate segment. (5 points)