Test #1C 100 points Math 200 Name:________________________________

Simplify your final answers. Show organized work. Defend all answers.

1) Find the mass of a wire with density $\delta = \frac{x+y^2}{\sqrt{1+\sin^2(x)}}$ grams per cm if it is located on the path $y = \cos(x)$ from the point $(0,1)$ to the point $(\pi,-1)$. (10 points)

2) Find and evaluate an integral representing the work done by $\vec{F} = \langle x+y, z, y \rangle$ on a particle that moves on the curve parameterized by $\vec{r}(t) = \langle \cos t, \sin t, t \rangle$ from $t = 0$ to $t = 2\pi$. (10 points)

3) Evaluate I = $\int_C \sqrt{y} - x\, dx + \frac{x-2}{y}\, dy$ if C is a line segment from (0, 1) to (3, 1) and a curve from (3, 1) to (5, 9) on $y = (x-2)^2$. (10 points)

4) Let $f = f(t)$ and $\vec{g} = \langle P, Q \rangle$ where P and Q are also functions of t. Prove $(\vec{g}f)' = \vec{g}'f + \vec{g}f'$. (5 points)

5A) Find all the second partial derivatives of $f(x, y) = \cos(xy)$. (6 points)

5B) What is $\nabla f\left(\frac{\pi}{2}, 1\right)$ if $f(x, y) = \cos(xy)$? (4 points)

6) Find the equation of the plane that contains the points P = $(1,1,0)$, Q = $(2,0,5)$, and R = $(0,1,1)$. Write your final answer in standard form. (10 points)

7) Let $\vec{a} = \langle 1,1,2 \rangle$, $\vec{b} = 2\hat{i} - 2\hat{j} + \hat{k}$, and $\vec{c} = \langle 1,0,1 \rangle$. (25 points; #7A is 4 points, the others are 3 points)

7A) Find the area of the parallelogram spanned by $\vec{b}$ and $\vec{c}$.

7B) Find the work done on an object with displacement $\vec{a}$ by force $\vec{b}$.

7C) Find the flux of $\vec{a}$ through the parallelogram spanned by $\vec{c}$ and $\vec{b}$, oriented from $\vec{c}$ to $\vec{b}$.

7D) Find the volume of the box spanned by $\vec{a}$, $\vec{b}$, and $\vec{c}$.

7E) Find the component of $\vec{a}$ onto $\vec{b}$.

7F) Find the projection of $\vec{a}$ onto $\vec{b}$.

7G) Find an equation for the osculating plane at the point $r(0) = (1,0,-1)$ if $\vec{r}'(0) = \vec{b}$ and $\vec{r}''(0) = \vec{c}$.

7H) Find the curvature at $r(0) = (1,0,-1)$ if $\vec{r}'(0) = \vec{b}$ and $\vec{r}''(0) = \vec{c}$. Simplify your answer.

8) Find the arc length of $\vec{r}(t) = \langle \sin t, \cos t, \cosh t \rangle$ from t = 0 to t = ln(3). (10 points)

9) Sketch the surface corresponding to $z = 4 - x^2 - y^2$. Label the axes. (5 points)

10) Use the contour plot of f(x, y) to estimate $f_y(3,5)$. Draw an appropriate segment. (5 points)