Test #1 100 points Math 200 Name:______________________________

Circle or box, and simplify your final answers. Show work or some other defense of your answers.

1) Find the following. (25 points)

a) Find the work done by the force $\vec{a} = \langle 1, 0, -1 \rangle$ over a displacement $\vec{b} = 2\hat{i} - \hat{j} - 2\hat{k}$.

b) Find the flux of $\vec{a} = \langle 1, 0, -1 \rangle$ through the parallelogram determined by $\vec{b} = 2\hat{i} - \hat{j} - 2\hat{k}$ and $\vec{c} = \overrightarrow{(1,1,1)}$.

c) Find the projection of $\vec{b} = 2\hat{i} - \hat{j} - 2\hat{k}$ onto $\vec{c} = \overrightarrow{(1,1,1)}$.

d) Find the component of $\vec{a} = \langle 1, 0, -1 \rangle$ on $\vec{b} = 2\hat{i} - \hat{j} - 2\hat{k}$.

e) Find the area of the parallelogram determined by $\vec{b} = 2\hat{i} - \hat{j} - 2\hat{k}$ and $\vec{c} = \overrightarrow{(1,1,1)}$.

f) Find the volume of the box determined by $\vec{a} = \langle 1, 0, -1 \rangle$, $\vec{b} = 2\hat{i} - \hat{j} - 2\hat{k}$, and $\vec{c} = \overrightarrow{(1,1,1)}$.

2) Find the equation of the plane that contains the points P = (1, 1, 1), Q = (3, 2, 2), and R = (1, 2, 1). Write your final answer in standard form. (10 points)

3) Find the arc length of $\vec{r}(t) = \left\langle \frac{t^2}{2}, \frac{t^3}{3}, \frac{t^2}{2} \right\rangle$ from $\left(\frac{1}{2}, \frac{-1}{3}, \frac{1}{2}\right)$ to $\left(\frac{9}{2}, 9, \frac{9}{2}\right)$. (10 points)

4) Use the product rule to help prove that $\hat{T}$ and $\hat{T}'$ are perpendicular. (5 points)

5) Find parameterizations for the following curves. Use either coordinate or vector notation for your final answer. Be sure to **state the domains of the parameters**. (25 points)

a) Parameterize the line segment from P = (1, 1, 1) to Q = (3, 2, 2).

b) Parameterize the circle in the plane z = 5 with center (3, 2, 5) and radius 3.

c) Parameterize $y + x^2 = x$ in the xy – plane from $(0,0)$ to $(3,-6)$.

d) Parameterize the curve formed by intersecting the cylinder $z^2 + y^2 = 4$ with the $x + 2y - z = 0$

e) Parameterize the ellipse $4x^2 + 9y^2 = 1$ in the xy – plane directed **clockwise**.

6) Find the following at P = (1, 2, 3) for the curve C if $\vec{r}(t)$ is a parameterization of C with $\vec{r}(0) = \langle 1,2,3 \rangle$, $\vec{r}'(0) = \langle 2,1,2 \rangle$, and $\vec{r}''(0) = \langle 4,0,5 \rangle$. (20 points)

a) Find the osculating plane. Write your answer in standard form.

b) Find κ, the curvature at P, and R, the radius of the osculating circle at P.

c) Find $\hat{T}$ and $\hat{N}$.

7) Let $\hat{u} = \langle u_1, u_2, u_3 \rangle$ be a unit vector and let the angles between $\hat{u}$ and $\hat{i}$, $\hat{u}$ and $\hat{j}$, and $\hat{u}$ and $\hat{k}$ be α, β, and γ respectively. (5 points)

a) Prove $u_1 = \cos\alpha$, $u_2 = \cos\beta$, and $u_3 = \cos\gamma$.

b) Use part a to prove $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$.